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Forecasting Stock Returns Using Option-Implied State Prices*

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Abstract

Options prices embed the risk preferences that determine expected returns in asset pricing models. Therefore, functions of options prices should predict returns. In this paper, we show that the State Prices of Conditional Quantiles (SPOCQ)—functions of options prices introduced in Metaxoglou and Smith (2016)—exhibit strong predictive ability for the U.S. equity premium. These SPOCQ series provide estimates of the market's willingness to pay for insurance against outcomes in various quantiles of the return distribution. They also relate to expected returns in prominent asset pricing models. Our SPOCQ series that captures relative risk aversion exhibits strong predictive ability for S&P 500 returns at horizons between 6 and 18 months, both in the full sample, 1990–2012, and out of sample. Our SPOCQ series that captures volatility aversion, however, exhibits no predictive ability due to the lack of skewness in the return distribution for the horizons considered.

JEL classification: C5, G12, G13

Key words: forecasting, options, pricing kernel, returns, state prices

A typical paper on stock return predictability proceeds by first proposing a variable that may drive the risk premium. This proposal may be motivated by a particular utility function, a particular specification for the dynamics of the state variables in the economy, or an accounting identity. The researcher then assesses whether the variable of choice predicts returns—see Lettau and Ludvigson (2010) or Rapach and Zhou (2013) for a succinct, yet informative, set of examples.

In this paper, we do not impose a model of optimizing behavior nor do we specify the distribution and dynamics of the state variables. Instead, we infer the risk premium and its

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components directly from options prices. We use State Prices of Conditional Quantiles (SPOCQ), which is a set of statistics that estimate at a point in time the discount rate applied to returns in a particular segment of the conditional distribution. Developed in Metaxoglou and Smith (2016), SPOCQ is an estimate of the price of an Arrow–Debreu security that would pay one dollar in the event that the return falls in, for example, the bottom quartile of the conditional distribution and zero, otherwise. The SPOCQ series are inherently forward looking because they use information from the derivatives markets and, hence, they are suitable candidate predictors for asset returns.

Metaxoglou and Smith (2016) document six features of SPOCQ for the S&P 500. First, left-tail returns have larger average SPOCQ than right-tail returns, which is consistent with standard notions of risk aversion. Second, traders discount top-quartile returns more heavily than third-quartile returns, which contradicts standard asset pricing models under risk aversion and is consistent with the pricing kernel puzzle (Jackwerth, 2000). Third, SPOCQ exhibit substantial month-to-month variation that appears to be noise rather than discount rate variation. Fourth, the left-tail SPOCQ peaked in months when major events occurred in the recent financial crisis. Fifth, increases in the dividend yield and the term spread are associated with clockwise rotations in the pricing kernel, which is consistent with increasing expected returns, whereas no such association exists for the default spread, CAY, consumer sentiment, or the Economic Policy Uncertainty Index. Finally, high volatility in stock returns is associated with lower state prices for bottom-quartile returns and higher state prices for top-quartile returns, which contradicts recent models that specify the pricing kernel as a linear function of squared returns. Overall, these findings suggest that SPOCQ-based statistics may predict stock returns.

In this paper, we derive the functions of SPOCQ that would forecast returns in three standard asset pricing models and then show that these functions have strong predictive ability for S&P 500 returns. We construct our SPOCQ series by evaluating the risk-neutral return distribution of the S&P 500 at the conditional quantiles of the physical return distribution. We use a mixture of logistic distributions that is both flexible and parsimonious to recover the risk-neutral distribution from the first derivative of the option pricing curves. We estimate the conditional quantiles using quantile regressions. By using quantiles, we avoid imposing a parametric structure on the shape of the conditional distribution.

Figure 1 shows our main SPOCQ-based predictor and the subsequent 18-month returns on the S&P 500. This predictor, which we term SPOCQD, is suggested by our theory. It captures the relative-risk-aversion component of the risk premium by measuring preferences for returns in the lower tail relative to the upper tail of the return distribution. SPOCQD equals the volatility of returns multiplied by the difference between a left- and a right-tail SPOCQ series. The left-tail SPOCQ series is the price of an Arrow–Debreu security that pays one dollar in the event that the return falls in the bottom quartile of the distribution, SPOCQ(0,25). The right-tail SPOCQ series is the price of an Arrow–Debreu security that pays one dollar in the event that the return falls in the upper quartile of the distribution, SPOCQ(75,100). While SPOCQ(0,25) captures aversion to downside risk for investors with long equity exposures, SPOCQ(75,100) captures aversion to upside risk for investors with short equity exposures.

We find that SPOCQD exhibits predictive ability in equity-premium regressions for horizons between 6 and 24 months. A one-standard-deviation increase in SPOCQD leads to an increase in annualized excess market returns between 6% and 8% for these horizons. SPOCQD is statistically significant based on *t*-statistics constructed using Hodrick (1992) standard errors to account for the overlapping nature of returns and it is not prone to the

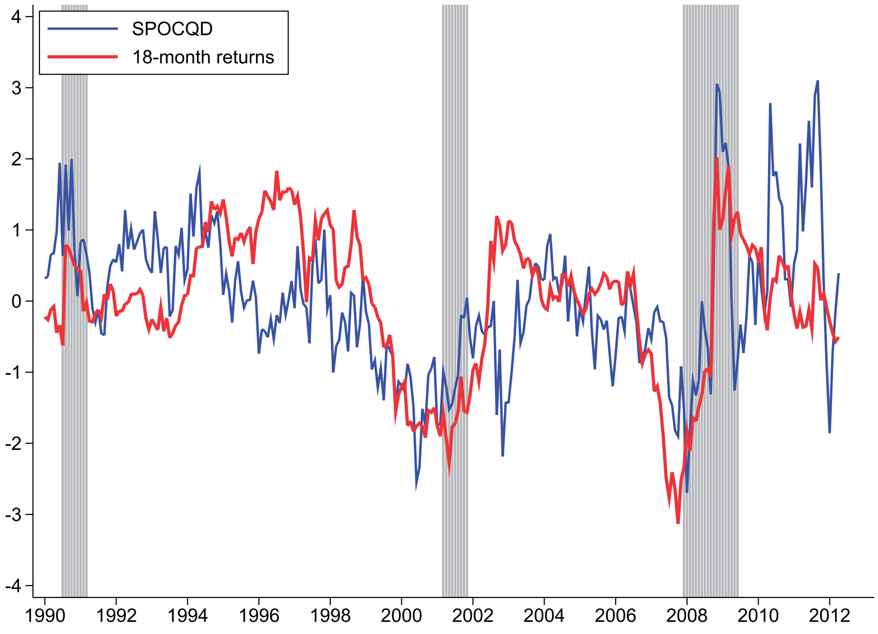

Figure 1. SPOCQD and subsequent 18-month excess returns.

Notes: To ease comparison, we have scaled both series to have zero mean and standard deviation one. The gray shaded areas indicate NBER recessions.

critique in Boudoukh, Richardson, and Whitelaw (2008). Through a series of bivariate regressions, we show that the predictive ability of SPOCQD is robust to the presence of popular predictors in the literature, including those in Goyal and Welch (2008). The only exception is the dividend yield. When paired with the dividend yield, SPOCQD is significant only at the 10% level for the 6-, 18-, and 24-month horizons. Moreover, using the Clark and West (2007) statistic, we show that SPOCQD exhibits significant out-of-sample (OOS) statistical performance at conventional levels. Importantly, SPOCQD exhibits OOS predictive ability using standard economic criteria, such as the Sharpe ratio and the certainty equivalent return (CER).

We also construct a SPOCQ-based predictor, which we term SPOCQS, to capture volatility aversion. Similar to SPOCQD, SPOCQS is suggested by our theory and is the market's willingness to pay to receive a dollar in the event that returns fall in either tail of the distribution. SPOCQS is the product of the sum of two SPOCQ series and the volatility of returns. The two SPOCQ series are SPOCQ(0,25) and SPOCQ(75,100). Volatility aversion, as captured by SPOCQS, surged in periods surrounding recessions, such as 1990–1991, 1998–2004, and 2007–2008. It plunged during 1995–1996 and 2005–2006, periods that coincide with the middle of two sustained bull markets. Unlike SPOCQD, SPOCQS fails to exhibit any predictive ability in the equity-premium regressions we considered. We attribute such a failure to the lack of substantial skewness in the S&P 500 returns.

The remainder of the paper is organized as follows. In Section 1, we provide an overview of SPOCQ and relate it to existing asset pricing models, as well as to the equity premium. In Section 2, we describe how we estimate the components of SPOCQ and discuss the salient feature of the SPOCQ series of interest during the period that is relevant for our analysis. We present the results of our forecasting exercises, including an evaluation of OOS statistical and economic performance, in Section 3. We present robustness check in Section 4, and conclusions follow in Section 5. The details for some of our derivations and some of our robustness checks are provided in the Appendix.

1 State Prices of Conditional Quantiles

1.1 Framework

In dynamic equilibrium models, the price of an asset, S_t equals the expected value of its discounted future payoff S_T

$$S_t = E_t\left[M_{t,T} S_T\right], \tag{1}$$

where $T > t$ and $M_{t,T}$ is the stochastic discount factor or pricing kernel between t and T.[1] The state of the economy at t can be described by a vector W_t, and $E_t[\cdot]$ is the expectation conditional on W_t. The researcher observes S_t and the prices of any derivatives defined by payoffs on the asset, but not W_t. Thus, we focus on the risk-neutral distribution implied by the observed asset and derivative prices, which is the risk-neutral distribution of returns on the asset after integrating out the unobserved component of the state space. Using the law of iterated expectations, the expression in Equation (1) becomes

$$1 = E_t\left[M_{t,T} R_{t,T}\right] = E_t\left[E_t\left[M_{t,T}|R_{t,T}\right] R_{t,T}\right] = \int E_t\left[M_{t,T}|R_{t,T}\right] R_{t,T} dF_t\left(R_{t,T}\right), \tag{2}$$

where $R_{t,T} \equiv S_T/S_t$ and $E_t\left[\cdot|R_{t,T}\right]$ is the expectation conditional on $\left\{W_t, R_{t,T}\right\}$. Multiplying and dividing by $E_t\left[M_{t,T}\right]$ produces

$$1 = E_t\left[M_{t,T}\right] \int \frac{E_t\left[M_{t,T}|R_{t,T}\right]}{E_t\left[M_{t,T}\right]} R_{t,T} dF_t\left(R_{t,T}\right) = E_t\left[M_{t,T}\right] \int R_{t,T} dF^*_{t,T}\left(R_{t,T}\right). \tag{3}$$

The risk-neutral conditional distribution of the asset return is given by

$$F^*_{t,T}(R) \equiv \int_{-\infty}^{R} M^*_{t,T}\left(R_{t,T}\right) dF_{t,T}\left(R_{t,T}\right) \equiv E_t\left[M^*_{t,T}|R_{t,T} \leq R\right] F_{t,T}(R), \tag{4}$$

where we define $M^*_{t,T}$ as

$$M^*_{t,T} \equiv M^*_{t,T}\left(R_{t,T}\right) \equiv \frac{E_t\left[M_{t,T}|R_{t,T}\right]}{E_t\left[M_{t,T}\right]}. \tag{5}$$

The steps in Equations (2)–(4) are similar to the projection of the pricing kernel onto the payoffs of a tradable asset in Engle and Rosenberg (2002). Therefore, $M^*_{t,T}$ can be labeled the projected pricing kernel. At time t, that is, taking the current state of the world (W_t) as given, $M^*_{t,T}(R)$ is the discount applied to the return outcome R.

1 Section 1.1 describes the framework for the SPOCQ statistic that was developed in Metaxoglou and Smith (2016). Our discussion overlaps significantly with the material in that paper and we include a shortened version here.

Equation (4) decomposes the risk-neutral conditional distribution into two components revealing the sources of its time variation. The first source is due to changes in the future return distribution $F_{t,T}(R)$. The second source is due to changes in the price of risk, $M^*_{t,T}(R)$. We focus on the second source, which we isolate by evaluating $F^*_{t,T}(R)$ at conditional quantiles of the asset returns. Specifically, we define the conditional quantile $q_{t,T}(\theta_j)$ such that $F_{t,T}(q_{t,T}(\theta_j)) = \theta_j$. The state price of the event $R_{t,T} \le q_{t,T}(\theta_j)$, which occurs with fixed probability θ_j, is then given by

$$F^*_{t,T}(q_{t,T}(\theta_j)) = E_t\left[M^*_{t,T}|R_{t,T} \le q_{t,T}(\theta_j)\right]\theta_j. \tag{6}$$

Equation (6) is an expression for a state price reflecting the market's willingness to pay for insurance against an event with a fixed probability.

We now define the SPOCQ

$$\mathrm{SPOCQ}_{t,T}(\theta_{j-1},\theta_j) = F^*_{t,T}(q_{t,T}(\theta_j)) - F^*_{t,T}(q_{t,T}(\theta_{j-1})). \tag{7}$$

Equivalently, using Equation (6), we can write SPOCQ as

$$\mathrm{SPOCQ}_{t,T}(\theta_{j-1},\theta_j) = \int_{q_t(\theta_{j-1})}^{q_t(\theta_j)} M^*_{t,T}dF_t(R_{t,T}) = (\theta_j - \theta_{j-1})E_t\left[M^*_{t,T}|\Re^{j-1,j}_{t,T}\right], \tag{8}$$

where $\theta_j > \theta_{j-1}$ and $\Re^{j-1,j}_{t,T}$ denote the states of the world at time T for which $q_{t,T}(\theta_{j-1}) \le R_{t,T} \le q_{t,T}(\theta_j)$. SPOCQ is the market's willingness to pay to receive a dollar in the event that the future return falls between the θ_{j-1} and θ_j quantiles.[2] It equals the probability of this event multiplied by the average of the projected pricing kernel conditional on this event. The time variation in SPOCQ is driven entirely by the willingness to pay for insurance against this event because the probability of the event is fixed. Under risk neutrality, this state price equals the probability of the event occurring

$$\mathrm{SPOCQ}^{RN}_{t,T}(\theta_{j-1},\theta_j) \equiv \theta_j - \theta_{j-1}. \tag{9}$$

Figure 2 illustrates how we obtain SPOCQ following a two-step approach. In the first step, we invert the physical distribution of returns to obtain the conditional quantiles $q_{t,T}(\theta_{j-1})$ and $q_{t,T}(\theta_j)$. In the second step, we evaluate the risk-neutral distribution at these conditional quantiles.

1.2 SPOCQ and Asset Pricing Models

In this section, we derive SPOCQ for two models in the asset pricing literature. The first model is a three-moment extension of the conditional CAPM. The second model is a discrete-time representative agent model with recursive preferences. Here and in the remainder of the paper, we use $T \equiv t+1$. Hence, we economize on notation by using $\mathrm{SPOCQ}_t(\theta_1,\theta_2)$, R_{t+1}, $q_t(\theta)$, and $\Re^{j-1,j}_{t+1}$ in place of $\mathrm{SPOCQ}_{t,T}(\theta_1,\theta_2)$, $R_{t,T}$, $q_{t,T}(\theta)$, and

2 For example, $\mathrm{SPOCQ}_{t,T}(0,25)$ is the market's willingness to pay to receive a dollar in the event that the future return falls in the bottom 25% of the distribution. Similarly, $\mathrm{SPOCQ}_{t,T}(75,100)$ is the market's willingness to pay to receive a dollar in the event that the future return falls in the top 25% of the distribution.

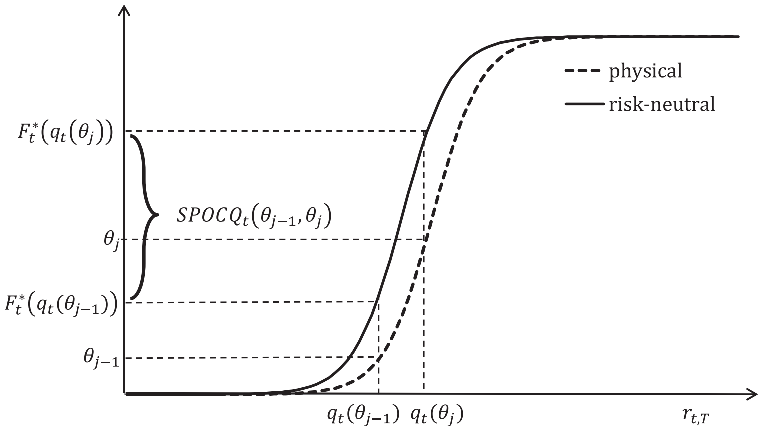

Figure 2. SPOCQ interpretation.

Notes: We obtain SPOCQ at a particular quantile (θ_j) by first inverting the physical return distribution to find the quantile $q_t(\theta_j)$ and then evaluating the risk-neutral distribution at $q_t(\theta_j)$.

$\Re_{t,T}^{j-1,j}$. In the empirical section of the paper, we use $\text{SPOCQ}_t(0, 25)$, $\text{SPOCQ}_t(25, 75)$, and $\text{SPOCQ}_t(75, 100)$ to predict expected returns, and, hence, our derivations pertain to these SPOCQ series.

Example 1: Three-Moment Conditional CAPM: The model follows Harvey and Siddique (2000) and allows the pricing kernel to be non-monotonic, an empirical regularity that has been labeled the pricing kernel puzzle (Jackwerth, 2000). In particular, the projected pricing kernel is

$$M_{t,t+1}^* = 1 - \beta_t(R_{t+1} - \mu_t) + \lambda_t\Big((R_{t+1} - \mu_t)^2 - \sigma_t^2\Big), \tag{10}$$

where $\mu_t \equiv E_t[R_{t+1}]$ and $\sigma_t^2 \equiv \text{var}_t[R_{t+1}]$. This expression is the same as equation (6) of Harvey and Siddique (2000) but it is parameterized such that $E_t\left[M_{t,t+1}^*\right] = 1$ holds. The first term in Equation (10) is the standard CAPM term and implies that the market discounts negative returns more heavily than positive returns. The second term implies an additional discount on large returns of either sign. If $\lambda_t = 0$ then the model reduces to the standard conditional CAPM. In addition, if R_{t+1} is conditionally normally distributed, then we show in Section A.1 of the Appendix that the following

$$\begin{aligned} \text{SPOCQ}_t(0, 25) &= 0.25 + 0.318\beta_t\sigma_t + 0.214\lambda_t\sigma_t^2 \\ \text{SPOCQ}_t(25, 75) &= 0.5 - 0.428\lambda_t\sigma_t^2 \\ \text{SPOCQ}_t(75, 100) &= 0.25 - 0.318\beta_t\sigma_t + 0.214\lambda_t\sigma_t^2. \end{aligned} \tag{11}$$

These expressions in Equation (11) reveal two sources of time variation in the SPOCQ series of interest in addition to the volatility of returns σ_t. The first is β_t and captures *relative risk aversion* because it represents the low value of a dollar in states of the world with high returns. The second is λ_t and captures *volatility aversion* because it represents the high value of a dollar in states of the world in which returns are far from the mean.[3] Note also that volatility aversion is increasing in σ_t^2 because high volatility implies that the events associated with returns in the lower and upper quartiles produce large changes in wealth. Hence, volatility-averse agents are willing to pay more for a dollar in the event that returns fall in the lower or upper quartiles of the distribution when volatility is high than when volatility is low. To separate the effects of β_t and λ_t, we write

$$\text{SPOCQ}_t(0,25) - \text{SPOCQ}_t(75,100) = 0.636\beta_t\sigma_t \tag{12}$$

$$\text{SPOCQ}_t(0,25) + \text{SPOCQ}_t(75,100) = 0.5 + 0.428\lambda_t\sigma_t^2. \tag{13}$$

An increase in β_t raises $\text{SPOCQ}_t(0,25)$ and lowers $\text{SPOCQ}_t(75,100)$. Thus, in the standard CAPM, the risk premium is determined by the market's willingness to pay for a dollar in the event that the return falls in the bottom quartile minus the market's willingness to pay for a dollar in the event that the return falls in the upper quartile of the distribution. The difference in Equation (12) captures the declining marginal utility of wealth due to the difference in the value of a dollar between low- and high-wealth states.

An increase in λ_t raises $\text{SPOCQ}_t(0,25)$ and $\text{SPOCQ}_t(75,100)$, which pertain to the tails of the distribution, relative to $\text{SPOCQ}_t(25,75)$, which pertains to the middle of the distribution. When λ_t is positive, the market is volatility-averse and is willing to pay a premium to receive a dollar in the event that returns are large in either direction.

Using Equations (10) and (12), the risk premium is proportional to the product of SPOCQ and conditional volatility and is given by

$$\begin{aligned} E_t\left[R_{t+1} - R_{t+1}^f\right] &= -\text{cov}_t\left[M_{t,t+1}^*, R_{t+1}\right] \\ &= \beta_t E_t\left[(R_{t+1} - \mu_t)^2\right] - \lambda_t E_t\left[(R_{t+1} - \mu_t)^3\right] \\ &= \beta_t\sigma_t^2 \\ &= (\text{SPOCQ}_t(0,25) - \text{SPOCQ}_t(75,100))\frac{\sigma_t}{0.636}, \end{aligned} \tag{14}$$

with $R_{t+1}^f = 1/E_t[M_{t,t+1}] - 1$ being the risk-free return. In this example, the third moment of returns equals zero because of the normality assumption. Thus, volatility aversion does not affect expected returns even if $\lambda_t \neq 0$. If returns were to exhibit skewness, then volatility aversion would affect expected returns.

Example 2: Representative Agent Models with Epstein–Zin Preferences: The model has been previously used in the literature by Bansal and Yaron (2004) and Bollerslev, Tauchen, and Zhou (2009), among others. Recursive preferences as in Epstein and Zin (1989) imply that the logarithm of the pricing kernel, $m_{t+1} \equiv \log(M_{t+1})$, is given by

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}, \tag{15}$$

3 Harvey and Siddique show that λ_t depends on the third derivative of the utility function. Note that a positive λ_t implies non-increasing absolute risk aversion, which is an important property of the preferences of a risk-averse individual.

where g_{t+1} denotes the log growth rate of aggregate consumption, $r_{a,t+1}$ is the log return on an asset that pays aggregate consumption as its dividend, and $\theta = (1-\gamma)(1-1/\psi)^{-1}$. The three preference parameters are the time discount factor δ, the intertemporal elasticity of substitution ψ, and the coefficient of risk aversion γ. When $\theta = 1$ the model collapses to the one with power utility.[4] Assuming that g_{t+1} is conditionally normally distributed, we show in Section A.2 of the Appendix the following

$$\begin{aligned} \mathrm{SPOCQ}_t(0,25) &= \Phi\left(-\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ \mathrm{SPOCQ}_t(25,75) &= \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} + 0.674\right) - \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ \mathrm{SPOCQ}_t(75,100) &= \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right), \end{aligned} \tag{16}$$

where $\Phi(\cdot)$ denotes the standard normal distribution function, $\sigma_t^{mr} \equiv \mathrm{cov}_t[m_{t+1}, r_{t+1}]$, $\sigma_t^r \equiv (\mathrm{var}_t[r_{t+1}])^{1/2}$, and r_{t+1} denotes the log return on the asset on which the pricing kernel is projected in Equation (4) to obtain SPOCQ. Noting that $\sigma_t^{mr}/\sigma_t^r \le 0$, we see that, in this model, SPOCQ depends on the covariance between the log pricing kernel and the log return. As this covariance becomes more negative, $\mathrm{SPOCQ}_t(0,25)$ and $\mathrm{SPOCQ}_t(25,75)$ increase, while $\mathrm{SPOCQ}_t(75,100)$ decreases.

To separate the effects of relative risk aversion and volatility aversion, we use the following

$$\mathrm{SPOCQ}_t(0,25) - \mathrm{SPOCQ}_t(75,100) \approx 0.636\beta_t\sigma_t^r \tag{17}$$

$$\mathrm{SPOCQ}_t(0,25) + \mathrm{SPOCQ}_t(75,100) \approx 0.5 + 0.214\left(\beta_t\sigma_t^r\right)^2, \tag{18}$$

where $\beta_t \equiv -\sigma_t^{mr}/\left(\sigma_t^r\right)^2$. We obtain the approximations from a second-order Taylor expansion of $\Phi(z)$ around the point $z = -0.674$.[5] These expressions are similar to their counterparts for the CAPM in Equations (12) and (13), except that the relative risk aversion term and the volatility aversion term are both driven by $\beta_t\sigma_t^r$. In the CAPM, the two terms were driven by two different parameters—β_t for risk aversion and λ_t for volatility aversion.

To give a more specific example, the long-run risk model of Bansal and Yaron implies

$$\beta_t = \frac{\left(\lambda_{m,\eta}\varphi_d + \lambda_{m,e}\beta_{m,e}\right)\sigma_t^2 + \lambda_{m,w}\beta_{m,w}\sigma_w^2}{\left(\varphi_d^2 + \beta_{m,e}^2\right)\sigma_t^2 + \beta_{m,w}^2\sigma_w^2}, \tag{19}$$

4 The pricing kernel in this model is a function of two variables, neither of which is the return on the asset under study. In contrast, the pricing kernel in the previous example was projected onto returns. This difference has no consequence for asset pricing because asset prices and returns are the same when the pricing kernel is projected onto returns as when it is not. Cochrane (2001) shows that this is a direct implication of the law of iterated expectation (see page 61).

5 Note that $\Phi^{-1}(0.25) = -\Phi^{-1}(0.75) = -0.674$.

where σ_t is the time-varying volatility of consumption. The rest of the parameters are related to preferences and the dynamics of consumption growth. Similarly, the model of Bollerslev et al. implies

$$\beta_t = \frac{-\gamma\sigma_{g,t}^2 - (1-\theta)\kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)q_t}{\sigma_{g,t}^2 + \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)q_t}, \tag{20}$$

where $\sigma_{g,t}^2$ denotes the volatility of consumption growth and q_t denotes the volatility of volatility that follows an AR(1) process.[6] In both models, dynamic components of consumption volatility change the risk premium by shifting mass between the left- and the right-tail SPOCQ. Although these models are rich in parameters, they affect SPOCQ only through the ratio $\sigma_t^{mr}/\sigma_t^r \equiv \beta_t\sigma_t^r$.

As in the three-moment CAPM of the first example, the risk premium is proportional to the product of SPOCQ and conditional volatility. From Bollerslev et al., and using their notation for the risk premium, we have

$$\begin{aligned} \pi_{r,t} &= -\mathrm{cov}_t[m_{t+1}, r_{t+1}] \\ &= \beta_t\left(\sigma_t^r\right)^2 \\ &\approx (\mathrm{SPOCQ}_t(0,25) - \mathrm{SPOCQ}_t(75,100))\frac{\sigma_t^r}{0.636}. \end{aligned} \tag{21}$$

These examples show that the relative risk aversion and volatility aversion components of the risk premium are parameterized separately in CAPM-type models and are encapsulated in a single ratio in the Epstein–Zin class of models. Both models assume a distribution for the state variable in the economy and a functional form for the pricing kernel, which is implied by the utility function. In the subsequent sections of the paper, we estimate SPOCQ non-parametrically—hence, we don't need to make assumptions about the distribution of the state variables or the functional form of the utility function.

1.3 SPOCQ and the Risk Premium

In this section, we show that the risk premium in a general asset pricing context can be approximated with a linear combination of SPOCQ multiplied by volatility. In combination with the results in the previous section, these derivations imply a forecasting model for returns and the risk premium. In particular, equilibrium in a representative agent economy implies that the risk premium is given by

$$\begin{aligned} E_t\left[R_{t+1} - R_{t+1}^f\right] &= -\mathrm{cov}_t\left[M_{t,t+1}^*, R_{t+1}\right] \\ &= -E_t\left[\left(M_{t,t+1}^* - 1\right)(R_{t+1} - \mu_t)\right] \end{aligned}$$

6 Recall that $\beta_t = -\mathrm{cov}_t(m_{t+1}, r_{t+1})/\mathrm{var}_t(r_{t+1})$. In the case of Bansal and Yaron, the numerator comes from their Equation (A14) and the denominator is their Equation (A13). In the case of Bollerslev et al., the numerator comes from their Equation (11) and the denominator comes from their Equation (12).

$$= -\sum_{j=1}^{J+1} (\theta_j - \theta_{j-1}) E_t \left[M^*_{t,t+1} (R_{t+1} - \mu_t) | \Re^{j-1,j}_{t+1} \right], \tag{22}$$

where R^f_{t+1} is the risk-free return and $\mu_t \equiv E_t[R_{t+1}]$. Additionally, we use $\theta_0 \equiv 0$ and $\theta_{J+1} \equiv 1$.[7] To show the connection between SPOCQ and the risk premium, we connect Equations (8) and (22) by decomposing $M^*_{t,t+1}$ into a piece that is linear in R_{t+1} and a remainder. Because the risk premium is the covariance between $M^*_{t,t+1}$ and R_{t+1}, only the linear relationship between them affects the risk premium.

We perform this decomposition for each segment of the return distribution given by $\Re^{j-1,j}_{t+1}$ separately using a linear projection of $M^*_{t,t+1}$ onto R_{t+1}. This projection provides the best linear predictor of $M^*_{t,t+1}$ as a function of R_{t+1} over $\Re^{j-1,j}_{t+1}$ and is given by

$$M^*_{t,t+1} = \alpha_{jt} \left(1 + \gamma_{jt} \left(R_{t+1} - \mu_{jt} \right) \right) + e_{j,t+1}. \tag{23}$$

The projection is defined such that the following hold:

$$E_t \left[e_{j,t+1} R_{t+1} | \Re^{j-1,j}_{t+1} \right] = 0 \tag{24}$$

$$E_t \left[M^*_{t,t+1} | \Re^{j-1,j}_{t+1} \right] = \alpha_{jt} \tag{25}$$

$$E_t \left[R_{t+1} | \Re^{j-1,j}_{t+1} \right] = \mu_{jt}. \tag{26}$$

Using Equation (23), we write the expectation term in Equation (22) as

$$\begin{aligned} E_t \left[M^*_{t,t+1} (R_{t+1} - \mu_t) | \Re^{j-1,j}_{t+1} \right] &= E_t \left[\left(\alpha_{jt} + \alpha_{jt} \gamma_{jt} \left(R_{t+1} - \mu_{jt} \right) + e_{j,t+1} \right) (R_{t+1} - \mu_t) | \Re^{j-1,j}_{t+1} \right] \\ &= \alpha_{jt} \left(\mu_{jt} - \mu_t + \gamma_{jt} \sigma^2_{jt} \right), \end{aligned} \tag{27}$$

where $\sigma^2_{jt} \equiv E_t \left[\left(R_{t+1} - \mu_{jt} \right)^2 | \Re^{j-1,j}_{t+1} \right]$. Substituting Equation (27) into Equation (22), the risk premium is

$$\begin{aligned} E_t \left[R_{t+1} - R^f_{t+1} \right] &= -\sum_{j=1}^{J+1} (\theta_j - \theta_{j-1}) \alpha_{jt} \left(\mu_{jt} - \mu_t + \gamma_{jt} \sigma^2_{jt} \right) \\ &= -\sum_{j=1}^{J+1} \text{SPOCQ}_t (\theta_{j-1}, \theta_j) \left(\mu_{jt} - \mu_t + \gamma_{jt} \sigma^2_{jt} \right). \end{aligned} \tag{28}$$

We obtain Equation (28) using $\alpha_{jt} = E_t \left[M^*_{t,t+1} | \Re^{j-1,j}_{t+1} \right]$, which implies that $\text{SPOCQ}_t(\theta_{j-1}, \theta_j) = (\theta_j - \theta_{j-1}) \alpha_{jt}$ based on Equation (8). In Equation (28), we express the risk premium as a weighted sum of SPOCQ that pertain to different intervals of the return distribution. The weights depend on the location term $(\mu_{jt} - \mu_t)$ and the scale term $(\gamma_{jt} \sigma^2_{jt})$. The location term

7 Given that r_{t+1} and R_{t+1} exhibit the one-to-one relationship, $r_{t+1} \equiv \ln(1 + R_{t+1})$, conditioning on $\Re^{j-1,j}_{t+1}$ is identical to conditioning on $\tilde{\Re}^{j-1,j}_{t+1} = \{ R_{t+1} | \exp(q_t(\theta_{j-1})) \leq 1 + R_{t+1} \leq \exp(q_t(\theta_j)) \}$.

captures the level of $M^*_{t,t+1}$ conditional on returns in the interval $\Re^{j-1,j}_{t+1}$, and the scale term captures the slope.

Using $\psi_j \approx -\left(\mu_{jt} - \mu_t + \gamma_{jt}\sigma^2_{jt}\right)/\sigma_t$, where σ_t is a measure of the dispersion in the conditional distribution of returns, we approximate the risk premium as follows:

$$E_t\left[R_{t+1} - R^f_{t+1}\right] \approx \sum_{j=1}^{J+1} \psi_j \text{SPOCQ}_t(\theta_{j-1}, \theta_j)\sigma_t. \tag{29}$$

The approximation in Equation (29) is exact if $\left(\mu_{jt} - \mu_t + \gamma_{jt}\sigma^2_{jt}\right)/\sigma_t$ is time invariant. If all the time variation in the conditional return distribution is captured by its first two moments, then the ratio $\left(\mu_{jt} - \mu_t\right)/\sigma_t$ would be time-invariant and depend only on j. In this case, there would be no approximation error due to the location term. As the number of segments J increases, the within-segment return variance decreases causing the scale term $\gamma_{jt}\sigma^2_{jt}/\sigma_t$ to decline toward zero. Thus, if most of the variation in the conditional return distribution is due to the first two moments and if the scale term is small, then the approximation error will be small.

Finally, note that Equation (29) suggests a linear regression of future excess returns on SPOCQ to estimate the risk premium. For quantiles below the median, the location term $(\mu_{jt} - \mu_t)$ will usually be negative, which implies a positive coefficient ψ_j on $\text{SPOCQ}_t(\theta_{j-1}, \theta_j)$ in such a regression. For quantiles above the median, the location term will be positive and the associated coefficient will be negative. Therefore, the risk premium is positively correlated with a left-tail SPOCQ and negatively correlated with a right-tail SPOCQ.

2 Estimating the Components of Spocq

2.1 Methods

To obtain SPOCQ, we need estimates of the risk-neutral distribution function $F^*_t(\cdot)$ and the conditional quantile $q_t(\theta)$ at which to evaluate this function. In both cases, we follow our work in Metaxoglou and Smith (2016). Hence, we provide only a brief overview of the estimation approach here.

We estimate the risk-neutral distribution directly from options prices (Breeden and Litzenberger, 1978). Using X and S_T to denote the strike and underlying price at the expiration date T, the price of a European put option may be written as

$$P_t(X,T) = E_t\left[M_{t,T} \times \max(X - S_T, 0)\right] = E_t\left[M_{t,T}\right]\int_{-\infty}^{X}(X - S_T)dF^*_t(S_T). \tag{30}$$

We define the adjusted put option price, also known as the forward option price, as

$$\tilde{P}_t(X,T) \equiv \frac{P_t(X,T)}{E_t\left[M_{t,T}\right]}, \tag{31}$$

and take the derivative with respect to the strike price to get

$$\frac{\partial \tilde{P}_t(X,T)}{\partial X} = \int_{-\infty}^{X} dF^*_t(S_T) = F^*_t(X). \tag{32}$$

Put-call parity produces a parallel expression for the adjusted call price as

$$\tilde{C}_t(X,T) \equiv \frac{C_t(X,T) - S_t + D_{t,T}}{E_t\left[M_{t,T}\right]} + X, \tag{33}$$

where $D_{t,T}$ denotes the present value of dividends to be paid before T. Differentiating yields

$$\frac{\partial \tilde{C}_t(X,T)}{\partial X} = F_t^*(X). \tag{34}$$

We obtain the risk-neutral distribution $F_t^*(S_T)$ by estimating the first derivative of the adjusted call and put option price curves, $\tilde{C}_t(X,T)$ and $\tilde{P}_t(X,T)$, with respect to X. We do so using a mixture of logistic distributions to approximate the risk-neutral distribution of the adjusted option prices in Equations (31) and (33).[8]

We use quantile-regression models to estimate conditional quantiles with volatility being the main explanatory variable focusing on the 25%, 50%, and 75% quantiles. In particular, we use the square root of *realized* continuous variation of daily returns for the S&P index over the last 20 trading days (CV) and the Chicago Board Options Exchange (CBOE) Volatility Index (VIX). The VIX includes a forward-looking component and a volatility-risk premium component, neither of which are in realized volatility. The first component is relevant for predicting quantiles, but the second is not. We allow the data to determine whether adding VIX improves the quantile predictions. Using $q_t(\theta) = x_t{}'\beta_\theta$, to denote the conditional $\theta-$quantile at time t, we have $x_t{}' = (1, \mathrm{CV}_t, \mathrm{VIX}_t - \mathrm{CV}_t)$. We estimate the quantile regressions using 268 observations on monthly returns between January 1990 and April 2012. Figure A1 provides the monthly time series of the conditional quantiles and the implied SPOCQ series.[9] Later in the paper, when we evaluate the OOS predictive ability of our SPOCQ series, we estimate the quantile regressions using an expanding-window approach.

2.2 SPOCQ Estimates

In Metaxoglou and Smith (2016), we showed that the SPOCQ series exhibit substantial month-to-month variation and that this variation is mostly noisy. To isolate the signal from the noise, we smooth the SPOCQ series using the fitted values from ARMA(1,1) models to construct a series of predictors in our equity premium regressions that follow. We estimate the ARMA(1,1) models using 268 observations between January 1990 and April 2012. Similar to the quantile regressions, when we evaluate the OOS predictive ability of our SPOCQ series, we estimate the ARMA(1,1) models using an expanding-window

8 Instead of fitting a distribution to the derivatives of the adjusted option prices, we fit the integral of a distribution to the adjusted options prices themselves. Fitting the curve before differentiating the option pricing curve avoids arbitrary assignment of the point at which the derivative applies. Thus, by using a mixture of logistic distributions, we are able to fit a flexible function to the adjusted option prices, and simultaneously impose the restriction that the derivative is a distribution. For additional details, see Section 3.1 and the Online Appendix in Metaxoglou and Smith (2016).

9 Our measure of continuous variation follows Bollerslev and Todorov (2011). For additional details, see Section 3.2 and the Online Appendix in Metaxoglou and Smith (2016).

approach.[10] These smoothed SPOCQ series, which are based on options data with time to expiration of one-month (28 days), are available in Figure A2.

In more detail, SPOCQ(0,25) indicates the investors' willingness to pay for insurance against outcomes in the lowest quantile of the return distribution. The insurance takes the form of a one-dollar payout in the event that returns fall in the bottom quartile. The difference between 1 and the SPOCQ(0,75) series, SPOCQ(75,100), indicates the willingness to pay for insurance against outcomes in the upper quartile. The difference between the SPOCQ(0,75) and SPOCQ(0,25) series, SPOCQ(25,75), indicates the willingness to pay for insurance against outcomes in the interquartile range of the return distribution.

Under risk neutrality, SPOCQ(0,25) would be equal to 0.25 and the SPOCQ(0,75) would be equal to 0.75. As a result, SPOCQ(25,75) would be equal to 0.5, and SPOCQ(75,100) would be equal to 0.25. Departures from these nominal levels implied by risk neutrality capture aversion to different notions of risk. In particular, SPOCQ(0,25) captures aversion to downside risk. Investors seeking protection against long equity exposures are willing to pay a downside risk premium. In contrast, SPOCQ(75,100) captures aversion to upside risk. Investors seeking protection against short equity exposures are willing to pay the upside risk premium. Between January 1990 and April 2012, the mean of SPOCQ(0,25) is 0.32, and the mean of SPOCQ(75,100) is 0.22. Therefore, investors' willingness to pay for insurance against outcomes in the lower quartile was higher than their willingness to pay for outcomes in the upper quartile of the return distribution.

The difference between SPOCQ(0,25) and SPOCQ(75,100) reveals the component of the risk aversion that emanates from relative risk aversion. When SPOCQ(0,25) is large relative to SPOCQ(75,100) the market places greater marginal value on a dollar in low-return states of the world relative to high-return states. Figure A2 shows that SPOCQ(0,25) exceeds SPOCQ(75,100) most of the time in our sample, except for 3 months in the second half of 2000.

The sum of SPOCQ(0,25) and SPOCQ(75,100) reveals volatility aversion, and it exceeds 0.5 for most of the sample. It averages 55 cents with a standard deviation of 2 cents. Some of the largest values of the sum of the two SPOCQ series occur during 1997–2003 and 2008–2009, which are periods of high volatility identified in Bloom (2009). The period 1997–2003 includes events such as the Asian Crisis (Fall 1997), the LTCM crisis (Fall 1998), September 11 (Fall 2001), the Enron/WorldCom scandal (Summer/Fall 2002), and Gulf War II (Spring 2003). The period 2008–2009 includes the most recent financial crisis. The 3 months between April and June 2007 are characterized by the lack of volatility aversion with the sum of SPOCQ(0,25) and SPOCQ(75,100) falling below 25 cents. This was a period of increasing stock prices and relatively low volatility—the first signs of the recent financial crisis started in August 2007.

3 Results

3.1 Predicting Market Returns

In this section, we compare the predictive performance of our state-price series with that of other predictors previously suggested in the literature. Using a sample of 268 monthly

10 In Section 4, we check the robustness of our findings to the smoothing of the SPOCQ series. For the use of smoothed predictors see Campbell and Thompson (2008), among others.

observations between January 1990 and April 2012, we estimate linear regressions of the form

$$r_{t+k} = a_{i,k} + x_{i,t}{}'\beta_{i,k} + \varepsilon_{i,t+k} \tag{35}$$

$$r_{t+k} \equiv 100 \times \zeta_k \times \left(\mathrm{SP}^{\mathrm{open}}_{t+k}/\mathrm{SP}^{\mathrm{close}}_t - 1\right) - r^f_t. \tag{36}$$

We use i to denote the model and k to denote the forecasting horizon. In addition, r_{t+k} is the close–open return on the S&P 500 using the closing value of the index on the options trading date t that is the date used to estimate our SPOCQ-based predictors, $\mathrm{SP}^{\mathrm{close}}_t$, and the opening value k days apart, $\mathrm{SP}^{\mathrm{open}}_{t+k}$. We use r^f_t to denote the risk-free rate. Additionally, $\zeta_k = 12, 4, 2, 1, 2/3,$ and 1/2 is an annualization factor reflecting the six alternative horizons (1, 3, 6, 12, 18, and 24 months) we consider. Also consistent with the alternative horizons considered, the values of k are 28, 84, 168, 336, 504, and 672 days, respectively.[11] We use the T-bill rate on date t as a proxy for the risk-free return r^f_t. For brevity, we refer to annualized excess returns calculated using Equation (36) as excess returns in the remainder of our discussion. The error term $\varepsilon_{i,t+k}$ follows an MA($k-1$) process under the null of no predictability ($\beta_{ik} = 0$) because of overlapping observations.

We estimate our forecasting regressions via ordinary least squares (OLS) and compute standard errors following Hodrick (1992) to address the overlapping nature of the error term. Ang and Bekaert (2007) have shown that the standard errors retain the correct size, even in small samples, and in the presence of multiple regressors. They also perform better than their Newey–West or the Hansen–Hodrick counterparts. We consider only parsimonious specifications because rich specifications can have very poor OOS performance, which is also of interest, due to in-sample overfitting. To ease comparisons, we standardize the predictors to have zero mean and standard deviation equal to one.

Our first set of predictors is motivated from our derivations in Section 1.3. Based on Equation (29), we first predict returns using the following three SPOCQ series

$$\mathrm{SPOCQ25}_t \equiv \mathrm{SPOCQ}_t(0, 25)\sigma_t \tag{37}$$

$$\mathrm{SPOCQ50}_t \equiv \mathrm{SPOCQ}_t(25, 75)\sigma_t \tag{38}$$

$$\mathrm{SPOCQ75}_t \equiv \mathrm{SPOCQ}_t(75, 100)\sigma_t, \tag{39}$$

where σ_t is the volatility of returns. Additional SPOCQ series would reduce the size of the scale term in Equation (28) and, hence, any approximation error due to the same term in Equation (29) at the expense of parsimony of the predictive regressions. We use the conditional interquartile range as our measure of return volatility σ_t because it is readily available from our conditional quantile model. In Section 4, we check the robustness of our findings to alternative measures of return volatility.

11 For example, the first observation for r_{t+k} is constructed using the index closing value on Friday, January 19, 1990, and the opening value on Friday, February 16, 1990, in the case of the one month ($k = 28$) horizon. In the same spirit, the excess return in the case of the 6 month horizon ($k = 168$) is calculated using the closing value on January 19, 1990, and the opening value on Friday, July 6, 1990. Similar reasoning applies for the remaining observations and horizons.

Motivated by the two examples in Section 1.2, we also estimate models that separate relative risk aversion from volatility aversion using the following two SPOCQ series

$$\text{SPOCQD}_t \equiv (\text{SPOCQ}_t(0, 25) - \text{SPOCQ}_t(75, 100))\sigma_t \tag{40}$$

$$\text{SPOCQS}_t \equiv (\text{SPOCQ}_t(0, 25) + \text{SPOCQ}_t(75, 100))\sigma_t. \tag{41}$$

Similar to the SPOCQ series in Figure A2, the predictors in Equations (37)–(41) are based on the smoothed versions of $\text{SPOCQ}_t(0, 25)$, $\text{SPOCQ}_t(25, 75)$, and $\text{SPOCQ}_t(75, 100)$ derived using conditional quantiles and the fitted values of ARMA(1,1) models based on 268 monthly observations. These five SPOCQ-based predictors are also based on options with time to expiration of 1 month (28 days).[12]

In addition to the SPOCQ series, the remaining predictors fall within two broad groups. The first group consists of the volatility-related variables in Bollerslev, Tauchen, and Zhou (2009). The second group consists of a series of popular macro/finance variables employed extensively in the return predictability literature.[13] Starting with the volatility-related variables, our first predictor is the variance risk premium defined as the difference between the implied and the realized variance (RV). Bollerslev et al. use the squared VIX index from the CBOE as their measure of implied variance (IV). They estimate RV using the sum of the 78 within-day five-minute squared returns during the normal trading hours 9:30–4:00 P.M. along with the squared close-to-open overnight return. We also include IV and RV among our predictors.

The log price-earnings ratio (Log(PE)) and the log dividend yield (Log(DY)) for the S&P 500 are the first two of our macro/finance predictors. We also include the default spread (DFSP), which is given by the difference between Moody's BAA and AAA bond yields. Our next predictor is the 3-month T-bill rate (TBILL). Additionally, we use the term spread (TMSP), the difference between the 10-year and the 3-month Treasury yields, and the stochastically detrended 3-month T-bill (RREL). Finally, we follow Lettau and Ludvigson (2001) and include CAY, the residual of the cointegrating relation for log consumption (C), log asset wealth (A), and log labor income (Y), among our predictors. In a subsequent section of the paper, we also compare the predictive performance of the 14 predictors in Goyal and Welch (2008) extended to April 2012 with that of our SPOCQ-based predictors.[14]

3.2 Predicting with SPOCQ

Table 1 shows the results from regressions of excess returns on three SPOCQ series in Equation (37)–(39), SPOCQ25, SPOCQ50, and SPOCQ75, for horizons between 1 and 24 months. The coefficients for SPOCQ25 and SPOCQ75 are positive and negative, respectively, which is consistent with our derivations in Section 1.3. The SPOCQ25 coefficient is

12 Computing SPOCQ-based predictors using options with longer horizons would require options data with expiration dates of 3–24 months. Although there are options with 1 month (28 days) to expiration for every single month in our sample, we are missing numerous months of data for options with longer time to expiration. This is especially the case for horizons exceeding 3 months (84 days).

13 See for example, Goyal and Welch (2008), Section 2.3. in Lettau and Ludvigson (2010), or Section 3.1 in Rapach and Zhou (2013), and the references therein.

14 The series are readily available from Amit Goyal's website at http://www.hec.unil.ch/agoyal/ (accessed on January 31, 2015).

Table 1. Return predictability with state prices I

Horizon	SPOCQ25		SPOCQ50		SPOCQ75		R^2
(month)	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat	
1	1.27	0.07	20.27	0.98	−20.53	−1.92*	0.02
3	4.10	0.26	16.52	0.95	−18.63	−1.83*	0.06
6	10.75	0.79	13.36	0.85	−21.75	−2.17**	0.15
12	16.43	1.57	4.07	0.32	−18.73	−1.92*	0.22
18	21.62	2.09**	−6.85	−0.70	−13.82	−1.75*	0.24
24	24.59	2.72***	−13.23	−1.70*	−10.67	−1.52	0.24

Notes: The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The three SPOCQ-based predictors are defined in Equations (37)–(39) in the main text. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012.

Table 2. Return predictability with state prices II

Horizon	SPOCQD		SPOCQS		R^2
(month)	Coeff.	*t*-stat	Coeff.	*t*-stat	
1	6.53	1.73*	0.73	0.13	0.02
3	5.72	1.65*	1.85	0.44	0.04
6	7.55	2.43**	2.16	0.63	0.13
12	7.45	2.54**	1.60	0.60	0.21
18	6.61	2.32**	0.79	0.34	0.23
24	5.89	2.28**	0.52	0.26	0.20

Notes: The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The two SPOCQ-based predictors are defined in Equations (40) and (41) in the main text. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations between January 1990 and April 2012.

significant at 5% level in the case of the 18- and 24-month horizons with values of 21.62 and 24.59, respectively. The SPOCQ75 coefficient is significant at 5% level in the 6-month regressions and has a value of −21.75. It is also significant at 10% level for the remaining horizons—excluding the 24-month one—with values between −20.53 and −13.82. The SPOCQ50 coefficient, however, fails to be significant for horizons of 1–18 months at conventional levels; it is significant at 10% in the 24-month regression with a value of −13.23. Although the R^2 values are small, not exceeding 0.15 for horizons less than 12 months, we see notable R^2 values of 0.24 for longer horizons.

In Table 2, we regress excess returns on SPOCQD and SPOCQS, which capture risk aversion and volatility aversion, respectively. Based on the derivations in Section 1.2, SPOCQD depends on relative risk aversion and the volatility of returns. However, any predictive power of SPOCQD is likely to be driven by relative risk aversion component because volatility is known to be a poor predictor of returns; see Yu and Yuan (2011) for an informative summary. Consistent with this literature, we show in the next section that volatility fails to predict returns. Moreover, in Metaxoglou and Smith (2016), we show that variation in stock return volatility is primarily idiosyncratic, meaning that it is unrelated to the pricing kernel and, hence, should not be priced.

The SPOCQD coefficient is positive for all six horizons considered with values between 5.72 and 7.55. Therefore, a one-standard-deviation increase in SPOCQD leads to a 6–8% increase in annualized excess returns, approximately, depending on the horizon considered. It is significant at 5% for horizons exceeding 3 months, and it is significant at 10% for horizons less than 6 months. It is important to emphasize that the SPOCQD coefficient does not increase monotonically with the horizon as it is often the case for highly persistent predictors with no predictive ability (Boudoukh, Richardson, and Whitelaw, 2008).

The SPOCQS coefficient, however, fails to be significant at conventional levels across all six horizons considered. Hence, we find no evidence that our measure of volatility aversion can predict excess returns. Our derivations in Section 1.2 show that if the conditional distribution of returns fails to exhibit substantial skewness then expected returns are unaffected by volatility aversion in the model of Harvey and Siddique (2000). As it was the case for the three SPOCQ series in Table 1, we see notable R^2 values in the rather tight range 0.20–0.23 for horizons exceeding 6 months when we use SPOCQD and SPOCQS to predict returns.

To summarize, based on the regressions in Table 1 and Table 2, our SPOCQ series SPOCQ25 and SPOCQ75, as well as their combination capturing relative risk aversion, SPOCQD, exhibit strong predictive ability, and more so for longer horizons. Our measure of volatility aversion, on the other hand, fails to exhibit any predictive ability. To keep the set of results and the associated robustness checks that follow manageable, the remainder of our discussion will focus on SPOCQD for horizons of 6–24 months. First, we will compare the performance of SPOCQD with that of other popular predictors in the literature. Second, we will use bivariate regressions to show that the strong predictive ability of SPOCQD remains intact when we pair SPOCQD with such predictors. Finally, we will show that SPOCQD also exhibits strong OOS predictive ability using both statistical and economic criteria.

3.3 SPOCQ and Other Predictors

Table 3 shows the results from a series of univariate regressions based on SPOCQD and the alternative predictors we considered. The SPOCQD coefficient is positive and statistically significant at 5% across all horizons with values between 5.90 (24 months) and 7.59 (6 months). Similar to Table 2, the SPOCQD coefficient estimates do not increase monotonically with the forecasting horizon. This is also true for R^2 that is between 0.12 (6 months) and 0.22 (18 months). The only other predictors that are statistically significant at 5% are the variance risk premium in the 6-month regressions, and the dividend yield in the regressions for horizons exceeding 6 months; their coefficients are 2.29, 5.71, and 6.08. The corresponding R^2 values are 0.05, 0.17, and 0.21. Overall, SPOCQD delivers higher R^2 values than the remaining predictors for horizons 6–18 months. The magnitude of the SPOCQD coefficient also exceeds that of the remaining predictors for these horizons.

Table 3. Return predictability with univariate regressions

Predictor	6 months			12 months		
	Coeff.	*t*-stat	R^2	Coeff.	*t*-stat	R^2
SPOCQD	7.59	2.43**	0.12	7.48	2.57**	0.20
CAY	3.62	1.17	0.03	4.29	1.41	0.07
DFSP	1.16	0.27	0.00	1.95	0.65	0.01
Log(DY)	5.03	1.62	0.05	5.54	1.85*	0.11
Log(PE)	−4.72	−1.42	0.05	−4.05	−1.26	0.06
RREL	4.64	1.29	0.04	4.99	1.41	0.09
TBLL	−0.23	−0.07	0.00	−0.56	−0.19	0.00
TMSP	0.41	0.14	0.00	2.35	0.82	0.02
IV	3.45	0.93	0.02	3.12	1.33	0.04
RV	0.72	0.18	0.00	1.77	0.73	0.01
VRP	4.71	2.29**	0.05	2.20	1.42	0.02
Predictor	18 months			24 months		
	Coeff.	*t*-stat	R^2	Coeff.	*t*-stat	R^2
SPOCQD	6.62	2.34**	0.22	5.90	2.29**	0.20
CAY	4.48	1.49	0.10	5.21	1.75*	0.15
DFSP	1.50	0.57	0.01	1.32	0.56	0.01
Log(DY)	5.71	1.96**	0.17	6.08	2.10**	0.21
Log(PE)	−3.50	−1.15	0.06	−3.74	−1.26	0.08
RREL	3.76	1.28	0.07	2.11	0.94	0.03
TBLL	−1.01	−0.37	0.01	−1.41	−0.55	0.01
TMSP	3.27	1.19	0.05	4.19	1.52	0.10
IV	2.18	1.09	0.02	1.49	0.84	0.01
RV	1.20	0.64	0.01	0.62	0.38	0.00
VRP	1.60	1.15	0.01	1.47	1.14	0.01

Notes: We report results by forecasting horizon in months. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012.

Table 4 shows bivariate regressions in which we pair SPOCQD with each of the 10 predictors used in our univariate regressions. If SPOCQD is unaffected by the presence of these predictors, then we would expect it to have a significant coefficient with a value that is similar to its univariate one. Indeed, the average of the SPOCQD coefficients along the 10 bivariate regressions is very close to the univariate SPOCQ coefficient for all horizons. In general, the conclusions regarding the predictive power for SPOCQD remain unaffected when we switch from univariate to bivariate predictive regressions.

The SPOCQD coefficient is significant at 5% across all four horizons with a couple of exceptions. These exceptions arise when we pair SPOCQD with the dividend yield for horizons exceeding 12 months. In these three cases, the SPOCQD *t*-statistics are 1.78 and 1.66.

Table 4. Return predictability with bivariate regressions

Predictor	6 months					12 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	7.29	2.28**	2.86	0.91	0.14	7.11	2.44**	3.55	1.17	0.25
DFSP	7.62	2.41**	−0.20	−0.05	0.12	7.37	2.40**	0.64	0.20	0.21
Log(DY)	6.78	1.96*	1.62	0.47	0.12	6.28	2.01**	2.37	0.74	0.22
Log(PE)	6.77	2.18**	−2.19	−0.65	0.13	6.93	2.40**	−1.46	−0.45	0.21
RREL	7.29	2.38**	4.10	1.18	0.16	7.15	2.55**	4.46	1.31	0.28
TBLL	7.80	2.49**	1.18	0.35	0.12	7.63	2.61***	0.82	0.27	0.21
TMSP	8.83	2.60***	−3.11	−0.96	0.14	7.78	2.64***	−0.75	−0.26	0.21
IV	7.24	2.27**	2.36	0.65	0.13	7.17	2.37**	2.05	0.81	0.22
RV	7.61	2.41**	−0.20	−0.05	0.12	7.37	2.47**	0.88	0.35	0.21
VRP	7.41	2.37**	4.41	2.10**	0.16	7.40	2.56**	1.90	1.24	0.22

Predictor	18 months					24 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	6.22	2.20**	3.83	1.28	0.30	5.42	2.11**	4.64	1.57	0.31
DFSP	6.57	2.24**	0.33	0.12	0.22	5.85	2.21**	0.29	0.12	0.20
Log(DY)	5.03	1.78*	3.18	1.09	0.26	3.80	1.66*	4.17	1.52	0.27
Log(PE)	6.18	2.17**	−1.19	−0.39	0.23	5.23	2.05**	−1.79	−0.60	0.21
RREL	6.38	2.33**	3.29	1.16	0.28	5.78	2.29**	1.69	0.78	0.21
TBLL	6.66	2.43**	0.20	0.08	0.22	5.84	2.37**	−0.35	−0.14	0.20
TMSP	6.33	2.44**	0.74	0.30	0.22	5.03	2.29**	2.18	0.91	0.22
IV	6.44	2.20**	1.22	0.56	0.23	5.81	2.18**	0.62	0.32	0.20
RV	6.57	2.28**	0.41	0.20	0.22	5.91	2.24**	−0.09	−0.05	0.20
VRP	6.57	2.35**	1.34	0.98	0.23	5.85	2.29**	1.23	0.97	0.20

Notes: We report results by forecasting horizon in months for bivariate regressions pairing SPOCQD with one of the predictors listed in the leftmost column. For each forecasting horizon, the first two columns are the coefficient estimates and the *t*-statistics for SPOCQD, whereas the next two columns are the coefficient estimates and the *t*-statistics for the alternative predictor. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012.

Pairing SPOCQD with the dividend yield also leads to somewhat lower SPOCQD coefficients compared with their univariate analogs for the 18- and 24-month horizons with values of 5.03 and 3.80 as opposed to 6.62 and 5.90. The variance risk premium is the only other predictor that is significant at 5% with a *t*-statistic of 2.10 and a coefficient of 4.41 for the 6-month horizon. The remaining predictors fail to be significant at conventional levels across all horizons. Pairing SPOCQD with the stochastically detrended T-bill rate and CAY leads to the largest R^2 values, between 0.28 and 0.31, observed for horizons exceeding 6 months.

Finally, although not reported here, we also performed trivariate regressions with SPOCQ25 and SPOCQ75 along with the predictors from Table 3. Consistent with our discussion in Section 1.3, SPOCQ25 always enters with a positive coefficient, and SPOCQ75 always enters with a negative coefficient. The SPOCQ25 coefficient is significant at 5% except when dividend yield enters the specification for horizons exceeding 12 months. For the 18- and 24-month horizons, the SPOCQ25 coefficient is significant at 10%. The SPOCQ75 is also statistically significant at 5% for the majority of the trivariate regressions across all horizons.[15]

3.4 OOS Performance

3.4.1 Preliminaries

In this section, we assess the predictive ability of SPOCQD by splitting our full sample into an estimation period and an evaluation period to conduct a pseudo OOS forecasting exercise. If SPOCQD is an unstable predictor that only works well for part of the sample, or its full-sample success is driven by a few outliers, then it will perform poorly in this exercise (Rossi, 2013).

We chose an estimation period to have enough data to obtain reliable estimates and our evaluation period that be long enough to be representative. We work with univariate models only and employ both the expanding- and rolling-window estimation approaches. For the expanding window, we use the first 200 observations for estimation and the remaining 68 observations for OOS evaluation. Hence, the evaluation period extends from September 2006 to April 2012. Our estimation sample begins in January 1990 and additional observations are used as they become available. The procedure is repeated until the full sample has been exhausted. Because the parameter estimates use only data through $t-1$, the procedure resembles the information set of an investor in real time. For the rolling-window estimator, we use a window of 200 observations, which has the same length as the smallest expanding window, dropping earlier observations as additional observations become available.

In particular, we divide the full sample of 268 observations in an initial estimation period of n_1 observations and an OOS evaluation period of n_2 observations. Forecasts of excess returns implied by model i for horizon k are generated for the n_2 observations using

$$\hat{r}_{i,t+k} = \hat{a}_{i,t,k} + x_{i,t}{}'\hat{\beta}_{i,t,k}, \tag{42}$$

where $\hat{a}_{i,t,k}$ and $\hat{\beta}_{i,t,k}$ are the intercept and slope estimates obtained using either the expanding- or the rolling-window approach with n_1 observations. In addition, we use only in-sample information to estimate both the quantile regressions we use to compute SPOCQD and the ARMA(1,1) model that we use to smooth it. Specifically, we use an expanding-window approach similar to the one for the forecasting regressions. Figure A3 provides the monthly time series of the conditional quantiles and the implied SPOCQ series using the expanding window approach (see also Figure A4).[16]

15 There are several instances for which SPOCQ75 is significant at 10% when it fails to be significant at 5%.

16 We also estimated both our quantile regressions and the ARMA(1,1) model using a rolling-window approach for a window of 200 observations and our results regarding the OOS performance of SPOCQD are very similar to the ones presented here.

3.4.2 Statistical Performance

We evaluate the OOS statistical performance of the univariate models considered using the OOS R^2 (R^2_{OS}) calculated as follows:

$$R^2_{\mathrm{OS},i,k} = 1 - \left(\mathrm{MSFE}_{i,k}/\mathrm{MSFE}_{0,k}\right). \tag{43}$$

$R_{\mathrm{OS},i,k}$ captures the improvement in the mean-squared forecast error (MSFE) that the predictive model i achieves relative to the historical average (HA), here labeled model 0, for horizon k. A negative R^2_{OS} implies that the predictor performs worse than setting forecasts equal to the HA.[17] We test the null hypothesis, $\mathrm{MSFE}_{0,k} \leq \mathrm{MSFE}_{i,k}$, against the alternative, $\mathrm{MSFE}_{0,k} > \mathrm{MSFE}_{i,k}$, which is equivalent to testing the null $R^2_{\mathrm{OS},i,k} \leq 0$ against the alternative, $R^2_{\mathrm{OS},i,k} > 0$. To perform the hypothesis testing, we employ the adjusted MSFE-t-statistic of Clark and West (2007), which we calculate using Newey–West standard errors with the number of lags equal to the forecasting horizon to account for any autocorrelation in the forecast errors.

Table 5 reports the R^2_{OS} and the p-value for the Clark-and-West statistic for the univariate regressions in Table 3, using either an expanding window (top panels) or a rolling window (bottom panels) for different forecasting horizons; the first OOS date is September 2006. The table conveys the message of Goyal and Welch (2008) that individual predictive regression forecasts fail to outperform the HA benchmark in terms of their MSFE across various horizons. This is the case for 6 out of 10 predictors, using either the expanding- or the rolling-window approach.

Using either approach, SPOCQD exhibits statistically significant OOS predictive ability for all horizons between 6 and 24 months. It performs best in the case of the 18-month horizon. In all cases, the p-value of the Clark-and-West statistic is 0.07 or smaller. The R^2_{OS} values are between 0.11 and 0.18 using either the rolling- or the expanding-window approach.

In the case of the expanding-window regressions, the SPOCQD R^2_{OS} values are higher than those for the other predictors for horizons less than 24 months. The term spread performs best in the case of the 24-month horizon with an R^2_{OS} of 0.24, which is much larger than that for SPOCQD, which is equal to 0.17. SPOCQD performs better than any other regressor for horizons less than 24 months in the rolling-window regressions too. Once again, the term spread performs best in 24-month horizons with an R^2_{OS} value of 0.22 compared with 0.17 for SPOCQD.

The variance risk premium is the only other predictor that also exhibits statistically significant OOS predictive ability for all forecasting horizons using either an expanding- or a rolling-window approach. However, the R^2_{OS} values for the variance risk premium are much smaller than those for SPOCQD. Other than the term spread and the variance risk premium, the IV also exhibits statistically significant OOS predictive ability in the 12-month horizon but fails to outperform SPOCQD.

Overall, based on the results reported in Table 5, SPOCQD exhibits substantial OOS predictive ability using statistical criteria. Its predictive ability is superior to that of popular

17 See Campbell and Thompson (2008) and the discussion on page 654 of Lettau and Ludvigson (2010) regarding "protection against overfitting/data mining" for the proper use of OOS tests in a forecasting framework.

Table 5. Univariate regressions OOS statistical performance

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.106	0.16	0.176	0.165	SPOCQD	0.012	0.008	0.065	0.054
CAY	0.025	0.08	0.139	0.187	CAY	0.254	0.188	0.178	0.138
DFSP	−0.063	−0.011	0.008	−0.040	DFSP	0.333	0.245	0.154	0.218
Log(DY)	0.051	0.098	0.165	0.202	Log(DY)	0.109	0.046	0.009	0.01
Log(PE)	−0.053	−0.180	−0.308	−0.380	Log(PE)	0.645	0.939	0.949	0.914
RREL	0.065	0.1	0.045	−0.010	RREL	0.193	0.222	0.239	0.404
TBLL	−0.018	−0.012	0.007	0.027	TBLL	0.582	0.583	0.166	0.094
TMSP	−0.003	0.051	0.138	0.236	TMSP	0.699	0.001	0.003	0.016
IV	0.032	0.063	0.047	−0.027	IV	0.053	0.046	0.064	0.653
RV	−0.059	0.021	−0.001	−0.069	RV	0.719	0.179	0.244	0.74
VRP	0.044	0.034	0.042	0.029	VRP	0.033	0.026	0.012	0.009
MIN	−0.063	−0.180	−0.308	−0.380	MIN	0.333	0.939	0.949	0.914
MAX	0.065	0.1	0.165	0.236	MAX	0.193	0.222	0.009	0.016
(a) expanding window, R^2_{OS}					(b) expanding window, p-value				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.108	0.168	0.18	0.169	SPOCQD	0.01	0.005	0.053	0.041
CAY	0.025	0.075	0.125	0.169	CAY	0.238	0.18	0.178	0.137
DFSP	−0.096	−0.034	−0.015	−0.071	DFSP	0.299	0.247	0.191	0.222
Log(DY)	0.068	0.102	0.166	0.178	Log(DY)	0.121	0.081	0.033	0.035
Log(PE)	−0.079	−0.225	−0.356	−0.434	Log(PE)	0.671	0.92	0.933	0.895
RREL	0.065	0.083	0.02	−0.025	RREL	0.224	0.254	0.283	0.46
TBLL	−0.030	−0.023	0.004	0.023	TBLL	0.6	0.641	0.197	0.1
TMSP	−0.011	0.049	0.135	0.217	TMSP	0.909	0.003	0.003	0.011
IV	0.011	0.054	0.036	−0.034	IV	0.176	0.034	0.051	0.7
RV	−0.063	0.018	−0.005	−0.074	RV	0.696	0.191	0.259	0.752
VRP	0.043	0.036	0.042	0.03	VRP	0.038	0.017	0.004	0.007
MIN	−0.096	−0.225	−0.356	−0.434	MIN	0.299	0.92	0.933	0.895
MAX	0.068	0.102	0.166	0.217	MAX	0.121	0.081	0.033	0.011
(c) rolling window, R^2_{OS}					(d) rolling window, p-value				

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months. In panels (a) and (b), we report results using an expanding window. In panels (c) and (d), we report results using a rolling window of 200 observations. The OOS R-squared (R^2_{OS}) measures the reduction in MSFE relative to the HA benchmark. We report the p-values of the MSFE-adjusted statistic of Clark and West (2007) discussed in the main text. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the R^2_{OS} and the associated p-value for predictors other than SPOCQD.

predictors in the literature for horizons less than 24 months using expanding- and rolling-window regressions. In the next section, we show that SPOCQD also exhibits substantial OOS predictive ability using economic criteria.

3.4.3 Economic Performance

In the previous section, we showed that SPOCQD exhibits strong OOS predictive ability of stock returns for horizons up to 18 months. In this section, we check whether this predictive ability can generate economic value to a risk-averse investor allocating her total wealth between stocks and a risk-free asset based on the forecasting regressions we estimate. In particular, we consider a mean–variance investor with relative risk aversion γ, who allocates a share $\omega_{i,k,t}$ of her wealth to stocks, and a share $\left(1-\omega_{i,k,t}\right)$ to the risk-free asset at time $t+k$ based on forecasts from the univariate model i with the share given by

$$\hat{\omega}_{i,k,t}=\frac{1}{\gamma}\left(\frac{\hat{r}_{i,t+k}}{\hat{\sigma}_{t+k}}\right), \tag{44}$$

where $\hat{\sigma}_{t+k}$ is a forecast of the variance of stock returns. Following Campbell and Thompson (2008), we estimate $\hat{\sigma}_{t+k}$ using the sample variance from a five-year rolling window of historical returns. During the OOS period, the average utility for our hypothetical investor forming her portfolio according to Equation (44) is given by

$$\hat{\nu}_{i,k}=\hat{\mu}_{i,k}-\frac{\gamma}{2}\hat{\sigma}_{i,k}^{2}, \tag{45}$$

where $\hat{\mu}_{i,k}$ and $\hat{\sigma}_{i,k}$ are the mean and variance of the returns for the portfolio formed on the basis of Equation (44). Alternatively, if the hypothetical investor relies on the HA forecasts to form her portfolio, we have

$$\hat{\omega}_{0,k,t}=\frac{1}{\gamma}\left(\frac{\bar{r}_{t+k}}{\hat{\sigma}_{t+k}}\right), \tag{46}$$

$$\hat{\nu}_{0,k}=\hat{\mu}_{i,k}-\frac{\gamma}{2}\hat{\sigma}_{i,k}^{2}. \tag{47}$$

We consider several measures of economic performance for the portfolio selection based on Equations (44)–(47); see Kostakis, Panigirtzoglou, and Skiadopoulos (2012), among others. The first measure is the Sharpe ratio. The second measure is the return loss net of transaction costs as in DeMiguel, Garlappi, and Uppal (2009)

$$\widehat{RL}_{i,k}=\frac{\hat{\mu}_{i,k}}{\hat{\sigma}_{i,k}}\hat{\sigma}_{0,k}-\hat{\mu}_{0,k}. \tag{48}$$

We calculate the expression in Equation (48) assuming a proportional transaction cost of 50 basis points for the risky asset and zero transaction costs for the risk-free asset. Our third measure is the utility gain, or CER, which is the fee that an investor is willing to pay to trade using the information in the forecast of model i relative to the information in the HA model

$$\widehat{\mathrm{CER}}_{i,k}=\left(\hat{\mu}_{i,k}-\frac{\gamma}{2}\hat{\sigma}_{i,k}^{2}\right)-\left(\hat{\mu}_{0,k}-\frac{\gamma}{2}\hat{\sigma}_{0,k}^{2}\right). \tag{49}$$

Finally, to gauge the degree of rebalancing required to form the portfolio based on model i, we first calculate the portfolio turnover $\widehat{\text{PT}}_{i,k}$ using the portfolio allocation weights in Equation (44) as in DeMiguel et al. We then calculate the relative portfolio, $\widehat{\text{RTP}}_{i,k} = \widehat{\text{PT}}_{i,k}/\widehat{\text{PT}}_{i,0}$. Hence, if $\widehat{\text{RPT}}_{ik}$ exceeds 1, the allocation weights based on the forecasting regression for the predictor under consideration exhibit much more variation than the weights based on the HA.

We report the measures of performance discussed above for the expanding- and rolling-window forecasting regressions, in Table 6 and Table 7, respectively. The Sharpe ratio and the return loss have been annualized. The CERs have been annualized and are expressed in percentage terms. In all cases, we assume a coefficient of relative risk aversion $\gamma = 5$. We provide the same measures for $\gamma = 2$ and $\gamma = 10$ in Tables A7–A10.

The Sharpe ratios for SPOCQD fall as the forecasting horizon increases using either an expanding- or a rolling-window approach. This is also the case for the default spread and the stochastically detrended T-bill. For the remaining predictors, the relationship between the Sharpe ratios and the forecasting horizon is less clear using either approach.

In Table 6 that pertains to the expanding-window approach, the Sharpe ratios for SPOCQD are between 0.19 (24 months) and 0.45 (6 months). The Sharpe ratios for the other predictors are between 0.11 for the dividend yield (12 months) and 0.35 for term spread (24 months). We also see several instances of negative Sharpe ratios. Overall, the Sharpe ratios for SPOCQD are notably higher than those for the other predictors for horizons less than 24 months.

The portfolio turnover for all predictors with the exception of CAY is much higher than its HA counterpart for the expanding-window regressions. The SPOCQD relative portfolio turnover generally increases with the forecasting horizon—from 18.59 (6 months) to 33.08 (24 months). The same pattern holds for the remaining predictors with the exception of the variance risk premium. In general, the higher transaction costs for the portfolios based on the predictors considered relative to the HA, measured by relative portfolio turnovers exceeding one, do not offset the extra risk-adjusted returns. The return-loss for SPOCQD is between 2.1% (24 months) and 2.9% (12 months) per annum, adjusting for transaction costs, and exceeds its counterpart for the remaining predictors except for the term spread for the 24-month horizon.

The CER comparison for the expanding-window regressions confirms the conclusion from the comparisons of Sharpe ratios. The SPOCQD CERs are smaller for longer horizons falling from 3.75% per annum for the 6-month horizon to 1.17% per annum for the 24-month horizon. They exceed the CERs for the other predictors for horizons shorter than 24 months.

The measures of economic performance for the rolling-window regressions are generally similar to those for the expanding-window for SPOCQD, which outperforms the remaining predictors for horizons between 6 and 18 months on the basis of the Sharpe ratio and the return loss. Using the CER as a criterion, SPOCQD outperforms the remaining predictors for horizons less than 18 months. It performs slightly worse than the term spread in the 18-month horizon with a CER of 2.0% compared with 2.03%. The relative portfolio turnover for SPOCQD is much higher than that for the remaining predictors and seems to put SPOCQD in a disadvantage relative to the other predictors for the longer horizons, especially the 24 months.

Table 6. Univariate regressions OOS economic performance: expanding window, ($\gamma = 5$)

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.45	0.35	0.28	0.19	SPOCQD	18.59	23.81	29.37	33.08
CAY	0.09	−0.00	0	−0.04	CAY	0.12	0.09	0.07	0
DFSP	0.18	0.05	−0.01	−0.02	DFSP	3.05	5.55	5.49	6.54
Log(DY)	0.1	0.11	0.17	0.16	Log(DY)	4.08	5.05	6.62	8.77
Log(PE)	−0.13	−0.27	−0.24	−0.19	Log(PE)	4.32	5.53	6.57	8.25
RREL	0.26	−0.02	−0.13	−0.15	RREL	2.27	3.12	3.25	3.39
TBLL	−0.17	−0.21	−0.09	0.02	TBLL	1.89	1.99	2.14	1.97
TMSP	−0.18	0.01	0.21	0.35	TMSP	1.75	3.89	7.5	12.61
IV	0.04	0.03	−0.02	−0.09	IV	8.2	10.19	10.76	10.11
RV	−0.15	0	−0.07	−0.12	RV	4.42	9.6	10.82	11.18
VRP	0.29	−0.02	−0.04	−0.04	VRP	25.16	16.8	17.02	17.3
MIN	−0.18	−0.27	−0.24	−0.19	MIN	0.12	0.09	0.07	0
MAX	0.29	0.11	0.21	0.35	MAX	25.16	16.8	17.02	17.3
(a) Sharpe ratio					(b) Relative portfolio turnover (RPT)				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.023	0.029	0.027	0.021	SPOCQD	3.75	2.87	1.95	1.17
CAY	0.01	0.011	0.01	0.005	CAY	1.58	1.46	0.98	0.6
DFSP	0.013	0.014	0.009	0.006	DFSP	1.94	1.41	0.71	0.45
Log(DY)	0.01	0.017	0.02	0.019	Log(DY)	0.73	1.21	1.39	0.8
Log(PE)	0.002	−0.003	−0.006	−0.006	Log(PE)	−2.36	−2.83	−2.84	−2.54
RREL	0.016	0.01	0.002	−0.003	RREL	2.16	1.34	0.45	−0.09
TBLL	0.001	−0.000	0.004	0.009	TBLL	0.8	0.25	0.27	0.47
TMSP	0	0.011	0.023	0.032	TMSP	0.01	1.05	1.79	2.09
IV	0.008	0.012	0.009	0.002	IV	0.79	0.73	0.39	−0.20
RV	0.001	0.011	0.005	−0.001	RV	0.46	1.16	0.37	−0.22
VRP	0.017	0.01	0.007	0.005	VRP	2.31	0.77	0.53	0.18
MIN	0	−0.003	−0.006	−0.006	MIN	−2.36	−2.83	−2.84	−2.54
MAX	0.017	0.017	0.023	0.032	MAX	2.31	1.46	1.79	2.09
(c) Return loss (RL)					(d) Certainty equivalent return (CER)				

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a hypothetical investor with mean–variance utility function and coefficient of relative risk aversion γ equal to 5. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.

Table 7. Univariate regressions OOS economic performance: rolling window, ($\gamma = 5$)

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.43	0.35	0.29	0.21	SPOCQD	14.7	21.99	29.85	36.78
CAY	0.09	0.08	0.07	−0.04	CAY	0.13	0.13	0.16	0
DFSP	0.14	0.05	−0.02	−0.02	DFSP	2.09	4.52	4.95	6.34
Log(DY)	0.17	0.18	0.24	0.21	Log(DY)	4.67	6.65	9.21	12.23
Log(PE)	−0.13	−0.26	−0.24	−0.18	Log(PE)	3.45	4.83	6.67	8.62
RREL	0.23	−0.04	−0.15	−0.17	RREL	1.77	2.58	2.96	2.93
TBLL	−0.17	−0.21	−0.06	0.04	TBLL	1.96	2	2.33	2.16
TMSP	−0.24	0.02	0.25	0.37	TMSP	2.05	3.73	7.82	13.25
IV	−0.02	0.03	−0.04	−0.09	IV	5.07	8.75	10.18	10.63
RV	−0.14	−0.01	−0.08	−0.12	RV	4.09	8.24	10.11	10.98
VRP	0.24	−0.04	−0.06	−0.04	VRP	17.11	13.43	14.44	16.49
MIN	−0.24	−0.26	−0.24	−0.18	MIN	0.13	0.13	0.16	0
MAX	0.24	0.18	0.25	0.37	MAX	17.11	13.43	14.44	16.49
(a) Sharpe ratio					(b) Relative portfolio turnover (RPT)				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.023	0.029	0.028	0.023	SPOCQD	3.61	2.85	1.99	1.15
CAY	0.011	0.015	0.014	0.005	CAY	1.63	1.46	0.98	0.61
DFSP	0.012	0.014	0.009	0.006	DFSP	1.87	1.46	0.73	0.47
Log(DY)	0.013	0.02	0.025	0.023	Log(DY)	0.27	0.92	1.5	0.3
Log(PE)	0.003	−0.003	−0.005	−0.005	Log(PE)	−3.43	−3.75	−3.54	−3.03
RREL	0.016	0.009	0	−0.004	RREL	2.12	1.26	0.35	−0.15
TBLL	0.001	0	0.006	0.011	TBLL	0.88	0.21	0.35	0.55
TMSP	−0.002	0.012	0.025	0.035	TMSP	−0.18	1.12	2.03	2.33
IV	0.006	0.012	0.008	0.001	IV	0.68	0.81	0.35	−0.20
RV	0.002	0.01	0.005	−0.001	RV	0.56	1.13	0.34	−0.24
VRP	0.016	0.009	0.006	0.005	VRP	2	0.7	0.46	0.19
MIN	−0.002	−0.003	−0.005	−0.005	MIN	−3.43	−3.75	−3.54	−3.03
MAX	0.016	0.02	0.025	0.035	MAX	2.12	1.46	2.03	2.33
(c) Return loss (RL)					(d) Certainty equivalent return (CER)				

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a hypothetical investor with mean–variance utility function and coefficient of relative risk aversion γ equal to 5. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.

In summary, using a mean–variance investor with a coefficient of relative risk aversion equal to 5, SPOCQD performs better than the alternative predictors considered based on standard measures of economic performance for horizons less than 24 months using both expanding and rolling-window forecasting regressions.

4 Robustness Checks

4.1 Full-Sample Results

For our baseline results, in Table 2 we employ smoothed versions of the $\text{SPOCQ}_t(0, 25)$, $\text{SPOCQ}_t(25, 75)$, and $\text{SPOCQ}_t(75, 100)$ series, obtained using the fitted values of quantile regressions and ARMA(1,1) models estimated once using all 268 monthly observations to isolate the signal from the month-to-month variation that is noise and carries no predictive content. In Table A1, we examine the robustness of our baseline results to the smoothing of the underlying SPOCQ series reporting the results from regressions based on the original series.

The SPOCQD coefficient remains positive and is now significant at 5% for horizons exceeding 6 months. It fails to be significant at conventional levels for shorter horizons. The SPOCQS coefficient is still not significant at conventional levels for all horizons. Furthermore, the SPOCQD coefficients are substantially smaller, especially for the shorter horizons, which is consistent with attenuation bias due to measurement error associated with the noise. A one-standard-deviation increase in SPOCQD now leads to an increase of around 3.5% in returns as opposed to 6–8% in the baseline case for horizons longer than 6 months. We also see a drop in R^2 values relative to the baseline case with the maximum value now being 0.07 as opposed to 0.23.

We also examine the robustness of our results in Table 2 to the measure of return volatility σ_t used in Equations (40) and (41). In our baseline results, σ_t is the interquartile range in the return distribution using the 25% and 75% conditional quantile estimates estimated once using all 268 monthly observations. In a series of robustness checks, we experimented with the following measures of return volatility: the square root of realized continuous variation in Metaxoglou and Smith (2016), realized standard deviation using daily and 5-minute returns, VIX, the conditional volatility series from the discrete Heston-Nandi GARCH in Christoffersen, Heston, and Jacobs (2013), and the Dynamic Volatility Dynamic Jump (DVDJ) GARCH in Christoffersen, Jacobs, and Orthanalay (2012). We report our findings using the DVDJ GARCH conditional volatility series in Table A2 noting that the results based on the other volatility series are highly similar.[18]

The use of the DVDJ GARCH volatility has no substantial implications for either the statistical significance or the magnitude of the SPOCQD coefficient relative to the baseline case—we do note, however, a small decrease in its magnitude. The most notable decrease is that for the 3-month horizon, where the SPOCQD coefficient decreases from 5.72 to 4.75. We also see a decrease in R^2 values relative to the baseline case with the maximum value now being 0.18 as opposed to 0.23.

In Table A3, we examine the robustness in the predictive ability of SPOCQD alone to smoothing and the use of the DVDJ GARCH volatility series. To ease comparison, we

18 We thank Peter Christoffersen, Kris Jacobs, and Chay Ornthanalai, for making their MATLAB code available to us.

report the baseline results from Table 3 in the first row. In the absence of smoothing, the SPOCQD coefficient remains positive and is significant at 5% for horizons of 12–24 months. Once again, we see a decrease in the magnitude of both the SPOCQD coefficients and the R^2 values relative to the benchmark case. The use of the DVDJ GARCH volatility has notably smaller implications for our conclusions regarding SPCOQD compared with smoothing. This is the case for both the magnitude of the SPOCQD coefficient and the R^2 values, which are smaller than their baseline counterparts, and more so for the shorter horizons of 6–12 months.

In Table A4, we examine the robustness of our bivariate regressions to the smoothing of the SPOCQ series. The sign of the SPOCQD coefficient remains positive. The coefficient fails to be significant at conventional levels in the case of the 6-month horizon. The SPOCQD coefficient also fails to be significant at 5% for horizons 12–24 months when we pair SPOCQD with one of the following four series: CAY, the dividend yield, the IV, and the RV. With the exception of the dividend yield, the SPOCQD coefficient is significant at 10% level. The pattern of a decrease in the magnitude of the SPOCQD coefficient and R^2 relative to the benchmark case that we saw in the univariate regressions also emerges in the bivariate regressions. Once again, the decrease is, on average, more pronounced in the shorter horizons.

Using the DVDJ GARCH volatility series in the bivariate regressions has a minor effect on our conclusions regarding SPOCQD in the bivariate regressions as it was the case in the univariate regressions (Table A5). Both the SPOCQD and R^2 values are smaller relative to the baseline case. Although the SPOCQD coefficient fails to be significant at 5% when we pair SPOCQD with the dividend yield across all horizons, it remains significant at 10% for the shorter horizons of 6 and 12 months.

Table A6 provides a final check of the SPOCQD predictive ability in the bivariate regressions. We now pair the SPOCQD series with each of the 14 predictors Goyal and Welch (2008) updated until April 2012. SPOCQD is not significant at 5% when it is paired with the dividend-price ratio and the dividend yield for all four horizons considered. However, it is significant at 10% in the case of the 6, 12, and 18 months, when paired with these two predictors. The range of the SPOCQD coefficients is generally comparable to the range of the SPOCQD coefficients in the baseline regressions in Table 4. The range of R^2_{OS} across models for each of the four horizons is also comparable to its counterpart for our baseline bivariate regressions.

4.2 OOS Results

In our OOS baseline results, we compared the performance of SPOCQD and the alternative predictors considered on the basis of statistical and economic criteria assuming a fixed fraction (68/268) of the total number of observations as OOS. We now check the robustness of our findings to this assumption for the statistical performance of SPOCQD using the OOS forecast tests in Rossi and Inoue (2012).

Specifically, we compute Rossi and Inoue's sup-type ($\mathcal{R}_T$) and average ($\mathcal{A}_T$) statistics, which equal the maximum and average of the Clark-and-West statistic over a range of possible OOS window sizes. Following the authors' recommendation, we impose symmetry by fixing their parameter $\overline{\mu} = 1 - \underline{\mu}$ and $\underline{\mu} = 0.15$, which implies OOS window sizes between 40 and 228 observations in our case. Hence, the largest OOS window starts in January

Table 8. Univariate regressions OOS statistical performance: Rossi and Inoue (2012) tests of robustness to OOS window size

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	2.309*	3.127**	3.780**	3.956**	SPOCQD	1.385	2.336	3.724	4.752*
CAY	2.339*	4.441**	5.255**	6.057**	CAY	3.007	4.758*	6.373**	7.868**
DFSP	1.83	2.581*	3.532**	4.118**	DFSP	2.576	3.416	5.760**	7.474**
Log(DY)	3.764**	5.905**	6.781**	6.907**	Log(DY)	2.8	5.922**	9.434**	10.018**
Log(PE)	4.180**	5.493**	5.930**	6.140**	Log(PE)	4.474*	6.366**	7.423**	8.547**
RREL	4.014**	5.188**	5.633**	4.711**	RREL	2.868	3.957*	5.146*	6.355**
TBLL	2.051*	3.104**	3.894**	4.371**	TBLL	2.439	4.655*	7.681**	8.461**
TMSP	2.528*	4.147**	5.380**	6.403**	TMSP	2.404	5.316**	7.489**	8.308**
IV	2.558*	3.710**	4.188**	4.146**	IV	2.222	3.484	4.545*	5.931**
RV	2.733*	2.523*	2.874*	4.148**	RV	2.232	2.629	3.637	5.186*
VRP	3.347**	3.166**	3.699**	3.970**	VRP	2.228	2.937	4.004*	5.000*
(a) Expanding window, $\mathcal{R}_T$					(b) Rolling window, $\mathcal{R}_T$				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQD	0.451	0.586	0.733	0.643	SPOCQD	−0.200	−0.135	0.122	0.117
CAY	0.264	0.922*	0.942*	1.393*	CAY	0.254	0.784	0.754	1.299*
DFSP	0.818	0.663	0.561	0.454	DFSP	1.161*	1.139*	1.338*	1.516*
Log(DY)	1.655**	2.393**	2.889**	2.778**	Log(DY)	1.359*	2.439**	3.420**	3.362**
Log(PE)	1.759**	1.845**	1.595**	1.499**	Log(PE)	1.786**	2.206**	2.297**	2.484**
RREL	2.219**	2.411**	2.360**	1.268*	RREL	1.500*	1.604*	1.637*	0.815
TBLL	0.101	0.417	0.995*	1.337*	TBLL	0.414	1.351*	2.387**	2.762**
TMSP	0.593	1.484**	2.398**	3.216**	TMSP	0.164	1.552*	2.853**	3.833**
IV	0.958*	1.344*	1.283*	0.756	IV	0.776	1.265*	1.237*	1.209*
RV	1.109*	0.553	0.451	0.644	RV	0.891	0.49	0.498	0.778
VRP	1.776**	0.837	0.834	0.909*	VRP	1.172*	0.343	0.369	0.582
(c) Expanding window, $\mathcal{A}_T$					(d) Rolling window, $\mathcal{A}_T$				

Notes: We report results by forecasting horizon in months. Cell entries are either the maximum or average Clark–West statistic over window sizes between 40 and 228 observations. Panel (a) contains the "sup-type" statistic ($\mathcal{R}_T$) for expanding window forecasting regressions, Panel (b) contains the "sup-type" statistic ($\mathcal{R}_T$) for rolling window forecasting regressions, Panel (c) contains the average statistic ($\mathcal{A}_T$) for expanding window forecasting regressions, and Panel (d) contains the average statistic ($\mathcal{A}_T$) for rolling window forecasting regressions. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). For additional details, see Section 4.2 in the main text.

2009, while the smallest starts in May 1993. As was the case with a single OOS window size, we treat the historical mean as the nested model and perform one-sided tests. To implement the test, we re-estimated our quantile regressions using an expanding-window approach with estimation windows consistent with the OOS window sizes implied by the choice of $\underline{\mu}$. Because an ARMA(1,1) model can be unstable in small samples, we smoothed the SPOCQD series using exponential smoothing with a smoothing parameter of $\omega = 0.2$.[19]

Using expanding-window forecasting regressions, the "single-window" Clark-and-West statistic for SPOCQD using 68 OOS observations is significant at 5% for horizons of 6 and 12 months, and it is significant at 7% for the longer horizons; see panel (b) of Table 5. Table 8 shows that SPOCQD exhibits significant predictive ability at 5% for horizons exceeding 6 months based on the $\mathcal{R}_T$ statistic. It also exhibits predictive ability at 10% for the 6-month horizon using the same statistic. Thus, there exists an OOS window size for which SPOCQD is significantly better than the null model.

For the rolling-window forecasting regressions, the single-window Clark–West statistic for SPOCQD has a p-value less than 6% for all horizons (panel (d), Table 5). Using the $\mathcal{R}_T$ statistic for a rolling window, SPOCQD fails to exhibit significant predictive ability at conventional levels across all horizons less than 24 months. SPOCQD fails to exhibit any predictive ability based on the $\mathcal{A}_T$ statistic using either the rolling or expanding window. Computing SPOCQD requires estimating quantile regressions, which can be imprecise in small samples. This explains why it performs well by the $\mathcal{R}_T$ statistic, but does not perform well by the $\mathcal{A}_T$ statistic. The $\mathcal{A}_T$ statistic averages across periods with short estimation windows, where SPOCQD does relatively poorly, and longer estimation windows, where SPOCQD performs well.

5 Conclusion

Options prices convey information about the market's willingness to pay for insurance against future outcomes for the underlying asset, and, hence, capture risk preferences that determine the equity premium in standard asset pricing models. Therefore, and consistent with the recent empirical asset pricing literature, functions of option prices should predict returns. We show that a novel set of statistics that are functions of options prices, the SPOCQ, developed in Metaxoglou and Smith (2016), exhibit strong predictive ability in the case of the U.S. equity premium. SPOCQ provide an estimate of the market's willingness to pay for insurance against outcomes in various quantiles of the return distribution and are directly related to expected returns in standard asset pricing models with higher moments or recursive preferences.

Two of the SPOCQ series we estimate are of particular interest in our forecasting exercises for the equity premium. The first, SPOCQD, captures relative risk aversion via the difference in willingness to pay for insurance against downside and upside risk, that is against returns in the lower and upper tail of the return distribution. The second series, SPOCQS, captures volatility aversion—the willingness to pay for insurance against events in either tail of the return distribution. It does so via the sum of SPOCQ series capturing the willingness to pay for insurance against downside and upside risk.

We find that SPOCQD is correlated with the risk premium predicting returns over horizons between 6 and 24 months. A one-standard-deviation increase in SPOCQD leads to a 6–8% increase in excess returns. Its predictive ability is robust to the presence of popular

19 The exponential smoother is $\hat{z}_{t+1|t} = \omega z_t + (1-\omega)\hat{z}_{t|t-1}$, where z_t is the SPOCQD value at time t. An ARMA(1,1) model of the form $z_t = \phi z_{t-1} + \epsilon_t - \theta\epsilon_{t-1}$ with ϵ_t being white noise implies the same one-step ahead forecast as the exponential smoothing with $\phi = 1$ and $\theta = 1-\omega$.

predictors in the literature, including those in Goyal and Welch (2008), as shown in a series of bivariate regressions. SPOCQD also exhibits both statistical and economic predictive ability OOS. We find no evidence, however, that SPOCQS can predict returns. Thus, although the path of SPOCQS clearly displays volatility aversion, our forecasting results indicate that S&P 500 returns do not display enough skewness for this volatility aversion to significantly affect average returns.

In the asset pricing literature, variables that forecast returns can be interpreted as indicators of the level of systematic risk. Our results indicate that SPOCQD is such a variable. Moreover, because the predictive ability of SPOCQD remains strong when we add a range of macro/finance variables to our forecasting regressions, we conclude that SPOCQD captures aspects of systematic risk that these variables fail to account for. However, our current work does not explain why the systematic risk captured by SPOCQD varies over time. Such an explanation would require a model that connects fluctuations in SPOCQD to fluctuations in measurable economic state variables, which is a future research direction worth pursuing.

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Appendix

A.1 SPOCQ for Three-Moment Conditional CAPM

We derive SPOCQ for the three-moment conditional CAPM of Harvey and Siddique (2000) as presented in Example 1 in Section 1.2. The projected pricing kernel is

$$M^*_{t,t+1} = 1 - \beta_t(R_{t+1} - \mu_t) + \lambda_t\Big((R_{t+1} - \mu_t)^2 - \sigma_t^2\Big), \tag{A1}$$

where $R_t \sim N(\mu_t, \sigma_t^2)$. Using $\phi(\cdot)$ to denote the standard normal PDF, the formulae for the mean and variance of a truncated normal distribution along with the fact that $q_t(0.25) = -0.674$ imply the following conditional moments:

$$E_t[R_{t+1}|r_{t+1} < q_t(0.25)] = \mu_t - \sigma_t \frac{\phi(0.674)}{0.25} \tag{A2}$$

$$E_t\Big[(R_{t+1} - \mu_t)^2 | r_{t+1} < q_t(0.25)\Big] = \text{var}_t[R_{t+1}|r_{t+1} < q_t(0.25)] + (E_t[R_{t+1}|r_{t+1} < q_t(0.25)] - \mu_t)^2 \tag{A3}$$

$$E_t\left[(R_{t+1}-\mu_t)^2|r_{t+1} < q_t(0.25)\right] = \sigma_t^2\left(1-\frac{0.674\phi(0.674)}{0.25}-\frac{\phi(0.674)^2}{0.25^2}\right)+\sigma_t^2\frac{\phi(0.674)^2}{0.25^2}$$
$$= \sigma_t^2\left(1-\frac{0.674\phi(0.674)}{0.25}\right). \tag{A4}$$

Applying the same formulae to $q_t(0.75)$ yields the projected pricing kernel for the three segments of return distribution

$$E_t\left[M^*_{t,t+1}|r_{t+1} < q_t(0.25)\right] = 1+\beta_t\sigma_t\frac{\phi(0.674)}{0.25}+\lambda_t\sigma_t^2\frac{0.674\phi(0.674)}{0.25} \tag{A5}$$

$$E_t\left[M^*_{t,t+1}|q_t(0.25) < r_{t+1} < q_t(0.75)\right] = 1-2\lambda_t\sigma_t^2\frac{0.674\phi(0.674)}{0.5} \tag{A6}$$

$$E_t\left[M^*_{t,t+1}|r_{t+1} > q_t(0.75)\right] = 1-\beta_t\sigma_t\frac{\phi(0.674)}{0.25}+\lambda_t\sigma_t^2\frac{0.674\phi(0.674)}{0.25}. \tag{A7}$$

Using the symmetry of the normal distribution, $\phi(-0.674) = \phi(0.674) = 0.318$, it follows from Equation (8) that

$$\begin{aligned} \text{SPOCQ}_t(0,25) &= 0.25+0.318\beta_t\sigma_t+0.214\lambda_t\sigma_t^2 \\ \text{SPOCQ}_t(25,75) &= 0.5-0.428\lambda_t\sigma_t^2 \\ \text{SPOCQ}_t(75,100) &= 0.25-0.318\beta_t\sigma_t+0.214\lambda_t\sigma_t^2. \end{aligned} \tag{A8}$$

A.2 SPOCQ for Representative-Agent Models with Epstein–Zin Preferences

We derive SPOCQ for the Epstein–Zin model presented in Example 2 in Section 1.2. Recursive preferences as in Epstein and Zin (1989) imply that the logarithm of the pricing kernel is

$$m_{t+1} = \theta\ln\delta-\frac{\theta}{\psi}g_{t+1}+(\theta-1)r_{a,t+1}, \tag{A9}$$

where g_{t+1} is the log growth rate of aggregate consumption, $r_{a,t+1}$ is the log return on an asset that pays aggregate consumption as its dividend, and $\theta = (1-\gamma)(1-1/\psi)^{-1}$. The three preference parameters are the time discount factor δ, the intertemporal elasticity of substitution ψ, and the coefficient of risk aversion γ. When $\theta = 1$ the model collapses to one with a power utility.

Assuming that g_{t+1} is conditionally normally distributed, applying the Campbell and Shiller (1988) approximation, and solving the model implies that log returns are conditionally normally distributed as in Bansal and Yaron (2004). Standard formulae for the conditional moments of a bivariate normal distribution

$$E_t[m_{t+1}|r_{t+1}] = E_t[m_{t+1}]+\frac{\sigma_t^{mr}}{(\sigma_t^r)^2}(r_{t+1}-E_t[r_{t+1}]) \tag{A10}$$

$$\text{var}_t[m_{t+1}|r_{t+1}] = \text{var}_t[m_{t+1}]-\frac{\sigma_t^{mr}}{(\sigma_t^r)^2}, \tag{A11}$$

where $\sigma_t^{mr} \equiv \mathrm{cov}_t[m_{t+1}, r_{t+1}]$, $\sigma_t^r \equiv (\mathrm{var}_t[r_{t+1}])^{1/2}$, and r_{t+1} denotes the log return for the asset on which we project the pricing kernel. It follows from the formula for the mean of a lognormal random variable that the projected pricing kernel is

$$\begin{aligned} M^*_{t,t+1} &= \frac{E_t\left[M_{t,t+1}|r_{t+1}\right]}{E_t\left[M_{t,t+1}\right]} \\ &= \exp\left(\frac{\sigma_t^{mr}}{\left(\sigma_t^r\right)^2}(r_{t+1} - E_t[r_{t+1}]) - 0.5\left(\frac{\sigma_t^{mr}}{\sigma_t^r}\right)^2\right). \end{aligned} \tag{A12}$$

The formulae for the truncated mean of a lognormal random variable imply

$$\begin{aligned} E_t\left[M^*_{t,t+1}|r_{t+1} > q_t(\theta_j)\right] &= \frac{E_t\left[M^*_{t,t+1}\right]}{1-\theta_j}\Phi\left(\frac{\frac{\sigma_t^{mr}}{\left(\sigma_t^r\right)^2}q_t(\theta_j) + \frac{\sigma_t^{mr}}{\left(\sigma_t^r\right)^2}E_t[r_{t+1}] + \left(\frac{\sigma_t^{mr}}{\sigma_t^r}\right)^2}{\frac{\sigma_t^{mr}}{\sigma_t^r}}\right) \\ &= \frac{1}{1-\theta_j}\Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - \Phi^{-1}(\theta_j)\right), \end{aligned} \tag{A13}$$

where we use the fact that $E_t\left[M^*_{t,t+1}\right] = 1$ and $\theta_j \equiv \Phi\left(\left(q_t(\theta_j) - E_t[r_{t+1}]\right)/\sigma_t^r\right)$, which implies $q_t(\theta_j) = \Phi^{-1}(\theta_j)\sigma_t^r + E_t[r_{t+1}]$. Now, using $\Phi^{-1}(0.25) = -0.674$ and $\Phi^{-1}(0.75) = 0.674$, it follows that SPOCQ for the three segments of the return distribution is

$$\begin{aligned} \mathrm{SPOCQ}_t(0,25) &= \Phi\left(-\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ \mathrm{SPOCQ}_t(25,75) &= \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} + 0.674\right) - \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ \mathrm{SPOCQ}_t(75,100) &= \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right). \end{aligned} \tag{A14}$$

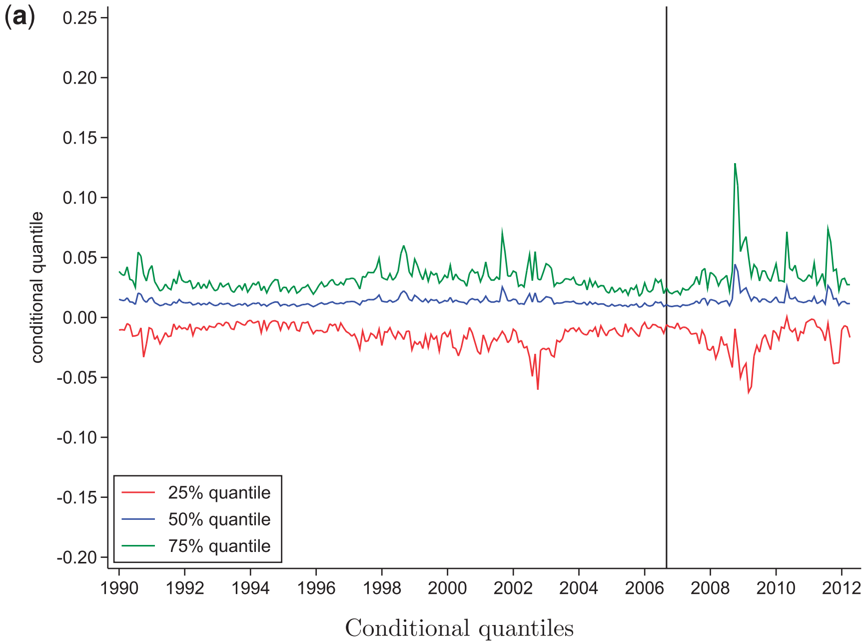
A second-order Taylor expansion of $\Phi(z)$ around $z = 0.674$ implies

$$\begin{aligned} \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} + 0.674\right) &\approx \Phi(0.674) + \phi(0.674)\frac{\sigma_t^{mr}}{\sigma_t^r} - \frac{1}{2}(0.674)\phi(0.674)\left(\frac{\sigma_t^{mr}}{\sigma_t^r}\right)^2 \\ &= 0.75 + 0.318\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.107\left(\frac{\sigma_t^{mr}}{\sigma_t^r}\right)^2. \end{aligned} \tag{A15}$$

It follows that

$$\begin{aligned} \mathrm{SPOCQ}_t(0,25) - \mathrm{SPOCQ}_t(75,100) &= \Phi\left(-\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) - \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ &\approx 0.636\frac{\sigma_t^{mr}}{\sigma_t^r} \end{aligned} \tag{A16}$$

$$\begin{aligned} \mathrm{SPOCQ}_t(0,25) + \mathrm{SPOCQ}_t(75,100) &= \Phi\left(-\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) + \Phi\left(\frac{\sigma_t^{mr}}{\sigma_t^r} - 0.674\right) \\ &\approx 0.5 + 0.214\left(\frac{\sigma_t^{mr}}{\sigma_t^r}\right)^2. \end{aligned} \tag{A17}$$



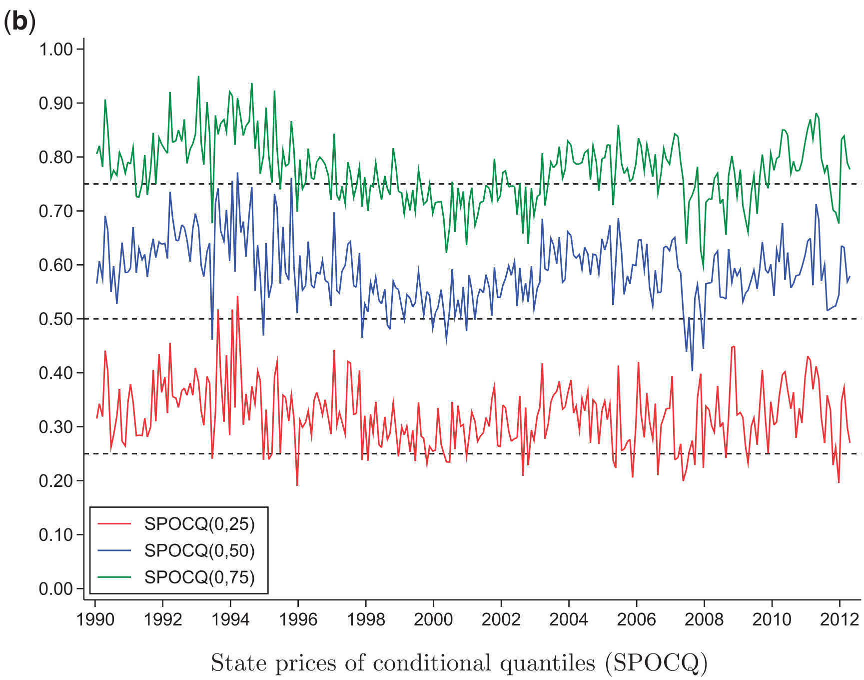


Figure A1. SPOCQ and conditional quantiles, full-sample fit.

Notes: Panel (a) shows the monthly time series of the conditional quantiles. Panel (b) shows the monthly time series of the implied SPOCQ. Each monthly observation corresponds to a trading date in our options data. For brevity, we write quantiles as integers.

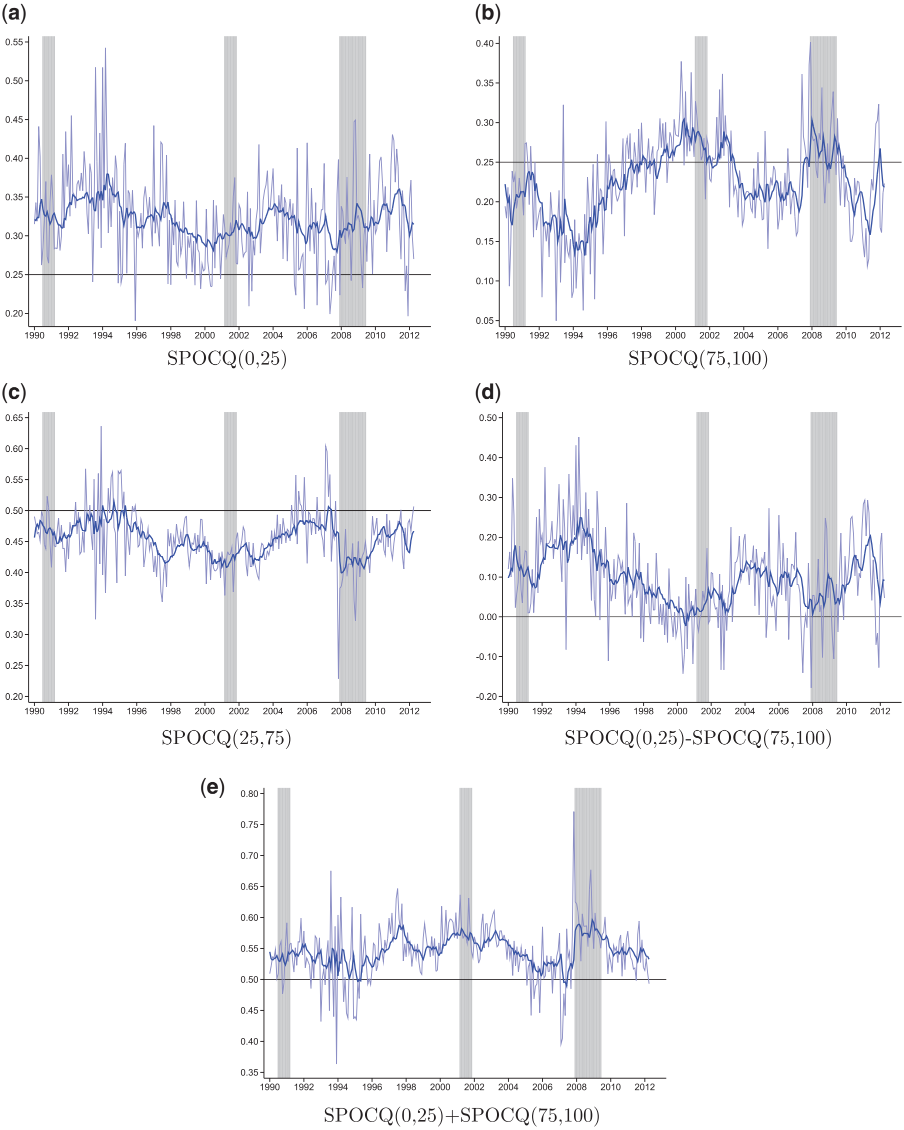


Figure A2. SPOCQ series, full-sample fit.

Notes: The light blue lines correspond to the original SPOCQ series. The dark blue lines correspond to the fitted values from an ARMA(1,1) model fitted in each of the original SPOCQ series for January 1990 to April 2012. The gray shaded areas in all panels indicate NBER recessions.

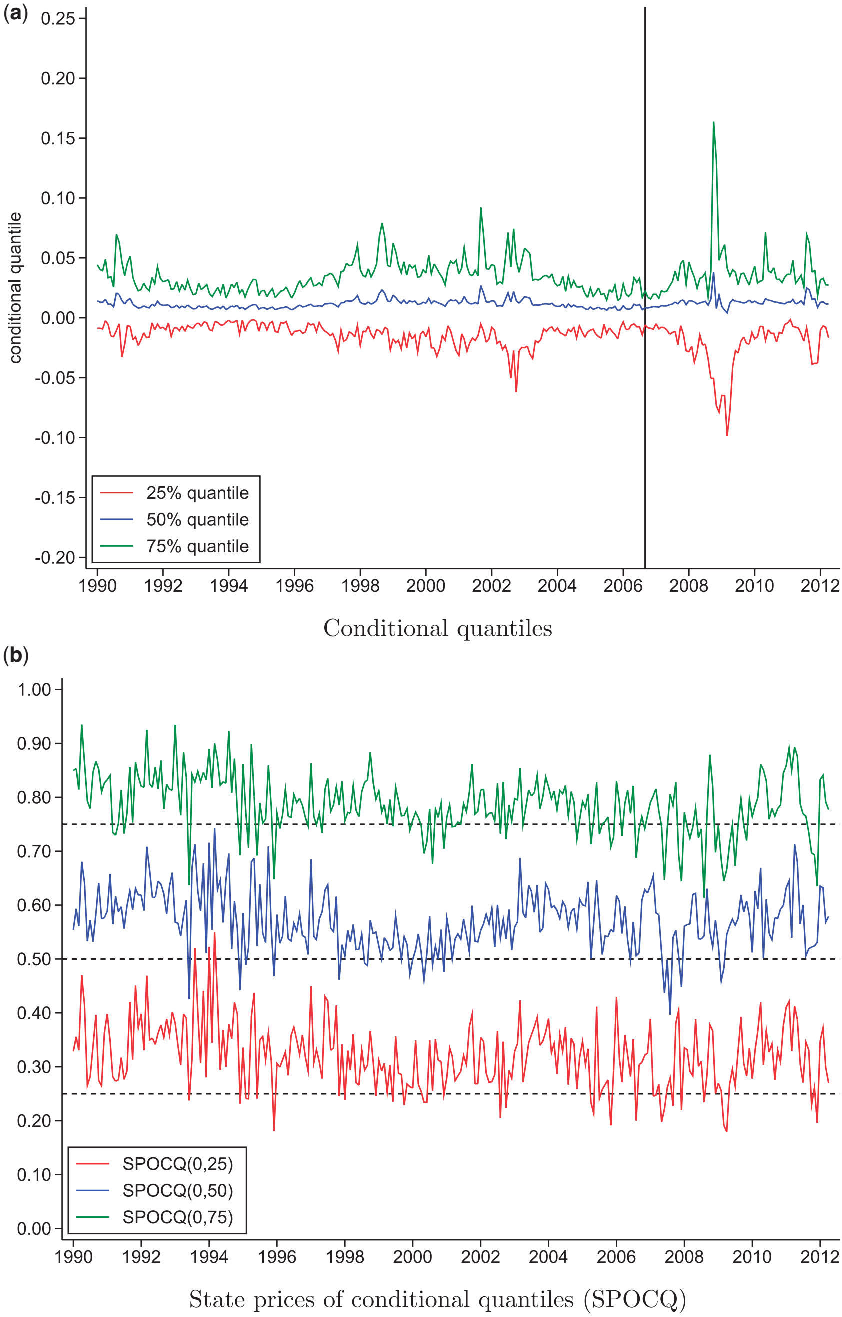


Figure A3. SPOCQ and conditional quantiles, expanding-window fit.

Notes: Panel (a) shows the monthly time series of the conditional quantiles. Panel (b) shows the monthly time series of the implied SPOCQ. Each monthly observation corresponds to a trading date in our options data. For brevity, we write quantiles as integers.

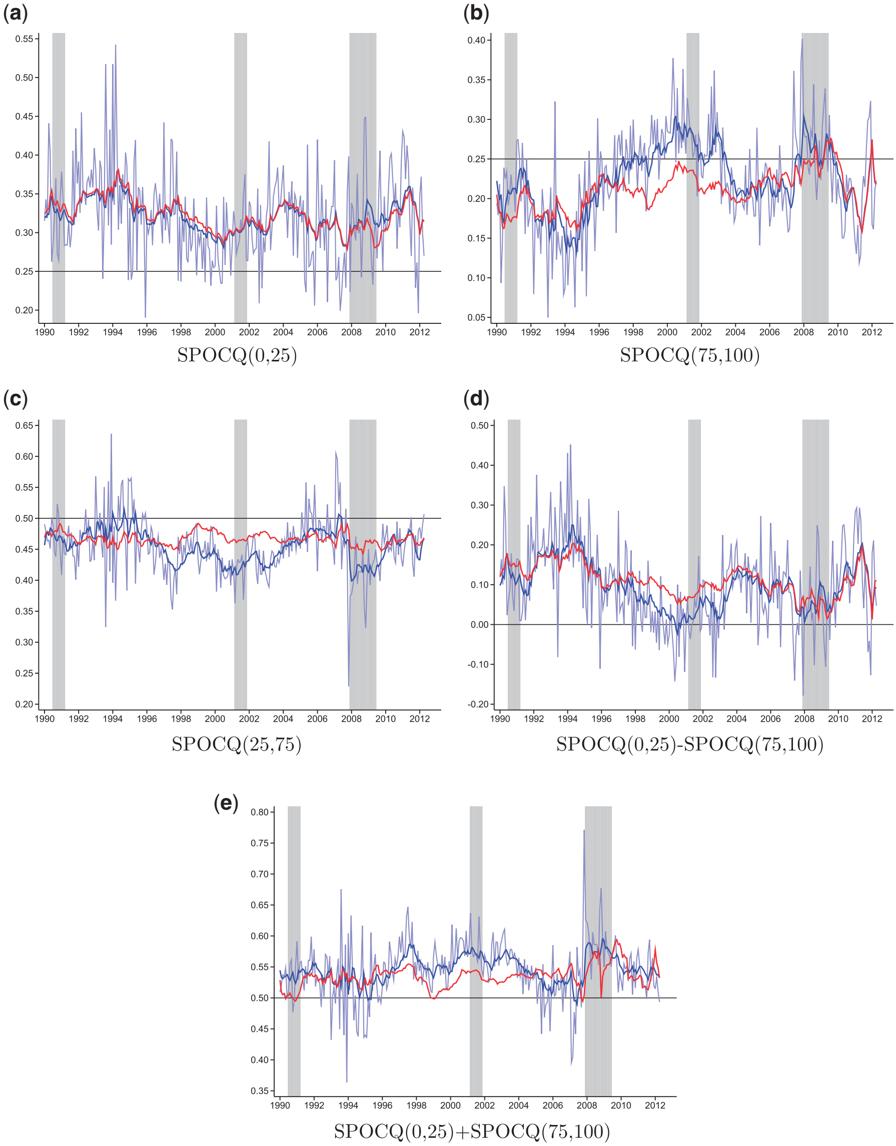


Figure A4. SPOCQ series, expanding-window fit.

Notes: The light blue lines correspond to the original SPOCQ series. The dark blue lines correspond to the fitted values from an ARMA(1,1) model fitted in each of the original SPOCQ series for January 1990 to April 2012. The red lines correspond to the fitted values from an ARMA(1,1) model using the expanding-window fit described in the text. The gray shaded areas in all panels indicate NBER recessions.

Table A1. Return predictability with state prices II: Robustness to smoothing of the SPOCQD and SPOCQS

Horizon	SPOCQD		SPOCQS		R^2
(month)	Coeff.	*t*-stat	Coeff.	*t*-stat	
1	1.81	0.34	2.09	0.36	0.00
3	1.95	0.68	1.94	0.47	0.01
6	2.03	0.93	2.45	0.71	0.02
12	3.55	2.05**	1.56	0.59	0.06
18	3.60	2.02**	0.74	0.32	0.07
24	3.35	2.07**	0.52	0.27	0.07

Notes: The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The two SPOCQ-based predictors are defined in Equations (40) and (41) in the main text. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations between January 1990 and April 2012.

Table A2. Return predictability with state prices II: Robustness to the use of the DVDJ GARCH volatility series

Horizon	SPOCQD		SPOCQS		R^2
(month)	Coeff.	*t*-stat	Coeff.	*t*-stat	
1	5.92	1.68*	0.03	0.01	0.01
3	4.75	1.46	0.43	0.10	0.03
6	6.84	2.33**	1.35	0.37	0.10
12	6.81	2.47**	0.67	0.23	0.17
18	6.00	2.23**	−0.02	−0.01	0.18
24	5.39	2.21**	−0.09	−0.05	0.16

Notes: The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The two SPOCQ-based predictors are defined in Equations (40) and (41) in the main text. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations between January 1990 and April 2012.

Table A3. Return predictability with univariate regressions: robustness to smoothing of SPOCQD and the use of the DVDJ GARCH volatility series

Model	6 months			12 months		
	Coeff.	*t*-stat	R^2	Coeff.	*t*-stat	R^2
Baseline	7.59	2.43**	0.12	7.48	2.57**	0.20
No smoothing	2.18	0.97	0.01	3.65	2.15**	0.05
DVDJ GARCH	6.87	2.34**	0.10	6.83	2.50**	0.17

Model	18 months			24 months		
	Coeff.	*t*-stat	R^2	Coeff.	*t*-stat	R^2
Baseline	6.62	2.34**	0.22	5.90	2.29**	0.20
No smoothing	3.65	2.10**	0.07	3.38	2.14**	0.06
DVDJ GARCH	6.00	2.25**	0.18	5.39	2.23**	0.16

Notes: We report results by forecasting horizon in months. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase in each variable. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012. The Baseline statistics are the ones from the first row of Table 3. The No smoothing statistics are based on univariate regressions without smoothing SPOCQD. The DVDJ GARCH statistics are based on univariate regressions using the conditional volatility series from a Dynamic Volatility with Dynamic Jumps (DVDJ) GARCH as in Christoffersen, Jacobs, and Orthanalay (2012).

Table A4. Return predictability with bivariate regressions: robustness to smoothing of SPOCQD

Predictor	6 months					12 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	1.91	0.82	3.47	1.10	0.03	3.34	1.91*	4.02	1.31	0.11
DFSP	2.08	0.98	0.91	0.21	0.01	3.47	2.09**	1.54	0.50	0.06
Log(DY)	0.71	0.35	4.81	1.58	0.05	2.16	1.59	4.87	1.66*	0.13
Log(PE)	1.35	0.64	−4.47	−1.37	0.05	3.00	2.03**	−3.49	−1.11	0.09
RREL	2.53	1.11	4.82	1.34	0.06	4.03	2.30**	5.28	1.48	0.15
TBLL	2.20	1.02	0.08	0.02	0.01	3.65	2.30**	−0.04	−0.01	0.05
TMSP	2.30	1.04	−0.34	−0.11	0.01	3.23	2.28**	1.30	0.46	0.05
IV	1.58	0.73	3.14	0.84	0.03	3.17	1.76*	2.51	1.01	0.07
RV	2.16	0.99	0.09	0.02	0.01	3.43	1.74*	0.77	0.29	0.05
VRP	3.26	1.47	5.36	2.65***	0.07	4.27	2.38**	3.05	1.88*	0.08

Predictor	18 months					24 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	3.31	1.88*	4.21	1.39	0.16	2.99	1.86*	4.97	1.66*	0.20
DFSP	3.52	2.04**	1.08	0.40	0.07	3.27	2.11**	0.94	0.40	0.07
Log(DY)	2.09	1.55	5.06	1.81*	0.19	1.67	1.57	5.57	2.01**	0.22
Log(PE)	3.10	2.04**	−2.92	−0.99	0.11	2.78	2.05**	−3.23	−1.11	0.12
RREL	3.94	2.20**	4.05	1.35	0.15	3.55	2.20**	2.37	1.04	0.10
TBLL	3.57	2.25**	−0.50	−0.19	0.07	3.25	2.29**	−0.94	−0.38	0.07
TMSP	2.89	2.18**	2.32	0.89	0.09	2.25	2.05**	3.45	1.31	0.12
IV	3.35	1.80*	1.54	0.71	0.08	3.21	1.89*	0.87	0.46	0.07
RV	3.60	1.77*	0.15	0.07	0.07	3.50	1.89*	−0.40	−0.21	0.06
VRP	4.13	2.21**	2.43	1.58	0.10	3.83	2.26**	2.23	1.57	0.09

Notes: We report results by forecasting horizon in months for bivariate regressions pairing SPOCQD with one of the predictors listed in the leftmost column without smoothing SPOCQD using an ARMA(1,1) model. For each horizon, the first two columns are the coefficient estimates and the *t*-statistics for SPOCQD, whereas the next two columns are the coefficient estimates and the *t*-statistics for the alternative predictor. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012.

Table A5. Return predictability with bivariate regressions: robustness to the use of the DVDJ GARCH volatility series

Predictor	6 months					12 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	6.57	2.18**	2.96	0.94	0.12	6.46	2.36**	3.65	1.21	0.22
DFSP	6.83	2.33**	0.27	0.06	0.10	6.69	2.37**	1.08	0.34	0.17
Log(DY)	5.80	1.80*	2.16	0.63	0.11	5.41	1.89*	2.86	0.90	0.19
Log(PE)	5.93	2.04**	−2.44	−0.72	0.11	6.18	2.31**	−1.67	−0.51	0.18
RREL	6.45	2.25**	3.95	1.13	0.13	6.37	2.46**	4.31	1.27	0.24
TBLL	7.02	2.40**	0.92	0.28	0.10	6.92	2.53**	0.57	0.19	0.17
TMSP	7.87	2.50**	−2.62	−0.81	0.11	6.95	2.52**	−0.32	−0.11	0.17
IV	6.63	2.24**	2.91	0.79	0.12	6.62	2.38**	2.58	1.06	0.19
RV	6.86	2.32**	0.13	0.03	0.10	6.72	2.43**	1.19	0.48	0.18
VRP	6.95	2.37**	4.82	2.33**	0.15	6.86	2.52**	2.31	1.49	0.19

Predictor	18 months					24 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
CAY	5.61	2.10**	3.92	1.31	0.26	4.92	2.03**	4.72	1.59	0.29
DFSP	5.91	2.16**	0.73	0.26	0.19	5.31	2.16**	0.64	0.26	0.17
Log(DY)	4.21	1.63	3.63	1.25	0.23	3.15	1.55	4.52	1.66*	0.25
Log(PE)	5.46	2.07**	−1.39	−0.46	0.19	4.64	1.98**	−1.96	−0.66	0.18
RREL	5.67	2.21**	3.16	1.11	0.23	5.22	2.21**	1.56	0.72	0.18
TBLL	6.00	2.31**	−0.02	−0.01	0.18	5.30	2.28**	−0.53	−0.22	0.16
TMSP	5.57	2.28**	1.12	0.45	0.19	4.43	2.15**	2.48	1.01	0.19
IV	5.86	2.16**	1.71	0.81	0.20	5.30	2.15**	1.06	0.57	0.17
RV	5.94	2.19**	0.69	0.35	0.19	5.38	2.18**	0.16	0.09	0.16
VRP	6.03	2.26**	1.70	1.21	0.20	5.41	2.24**	1.55	1.20	0.18

Notes: We report results by forecasting horizon in months for bivariate regressions pairing SPOCQD with one of the predictors listed in the leftmost column using the conditional volatility series from a Dynamic Volatility with Dynamic Jumps (DVDJ) GARCH as in Christoffersen, Jacobs, and Orthanalay (2012). For each horizon, the first two columns are the coefficient estimates and the *t*-statistics for SPOCQD, whereas the next two columns are the coefficient estimates and the *t*-statistics for the alternative predictor. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012.

Table A6. Return predictability with bivariate regressions: robustness to the use of Goyal and Welch regressors

Predictor	6 months					12 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
log(DP)	7.11	1.95*	0.88	0.24	0.12	6.49	1.88*	1.79	0.51	0.21
log(DY)	6.76	1.90*	1.53	0.42	0.12	6.31	1.91*	2.17	0.63	0.22
log(EP)	7.89	2.56**	−1.43	−0.50	0.12	7.78	2.73***	−1.43	−0.55	0.21
log(DE)	7.27	2.28**	1.65	0.57	0.13	7.08	2.34**	2.10	0.80	0.22
SVAR	7.66	2.39**	−0.46	−0.11	0.12	7.33	2.43**	1.01	0.38	0.21
BM	8.01	2.55**	−0.75	−0.23	0.12	7.75	2.68***	−0.49	−0.16	0.20
NTIS	6.77	2.05**	3.67	0.77	0.15	6.65	2.45**	3.71	0.86	0.25
TBL	7.81	2.50**	1.19	0.35	0.12	7.65	2.61***	0.91	0.30	0.21
LTY	7.65	2.41**	−0.57	−0.20	0.12	7.40	2.54**	0.76	0.28	0.21
LTR	7.44	2.40**	1.17	1.09	0.12	7.45	2.51**	0.23	0.31	0.20
TMS	8.72	2.59***	−2.86	−0.88	0.13	7.74	2.59***	−0.66	−0.22	0.21
DFY	7.60	2.40**	−0.06	−0.01	0.12	7.36	2.41**	0.70	0.22	0.21
DFR	7.80	2.42**	2.34	1.10	0.13	7.65	2.61***	1.84	1.03	0.22
INFL	7.51	2.43**	−2.29	−1.00	0.13	7.39	2.55**	−2.64	−2.33**	0.23

Predictor	18 months					24 months				
	SPOCQD		Predictor		R^2	SPOCQD		Predictor		R^2
	Coeff.	*t*-stat	Coeff.	*t*-stat		Coeff.	*t*-stat	Coeff.	*t*-stat	
log(DP)	5.13	1.65*	2.71	0.85	0.25	3.78	1.50	3.86	1.32	0.25
log(DY)	5.02	1.68*	2.97	0.96	0.25	3.68	1.52	4.13	1.43	0.26
log(EP)	6.92	2.44**	−1.37	−0.55	0.23	6.26	2.42**	−1.68	−0.78	0.21
log(DE)	6.14	2.15**	2.50	1.03	0.25	5.26	2.07**	3.34	1.70*	0.26
SVAR	6.54	2.24**	0.55	0.26	0.22	5.90	2.22**	−0.01	−0.00	0.20
BM	7.00	2.60***	−0.68	−0.25	0.22	6.16	2.61***	−0.46	−0.17	0.20
NTIS	5.95	2.28**	3.02	0.82	0.27	5.26	2.17**	2.89	0.86	0.24
TBL	6.66	2.43**	0.19	0.07	0.22	5.83	2.37**	−0.37	−0.15	0.20
LTY	6.52	2.29**	1.01	0.42	0.23	5.77	2.23**	1.26	0.53	0.20
LTR	6.70	2.34**	−0.57	−0.98	0.22	5.90	2.26**	0.02	0.04	0.20
TMS	6.25	2.40**	0.95	0.37	0.23	5.00	2.27**	2.27	0.93	0.22
DFY	6.57	2.25**	0.35	0.13	0.22	5.84	2.21**	0.35	0.14	0.20
DFR	6.79	2.36**	1.79	1.23	0.24	6.03	2.31**	1.36	1.24	0.21
INFL	6.57	2.34**	−1.61	−1.65*	0.24	5.86	2.28**	−1.35	−2.26**	0.21

Notes: We report results by forecasting horizon in months for bivariate regressions pairing SPOCQD with one of the predictors from Goyal and Welch (2008) updated until April 2012. For each horizon, the first two columns are the coefficient estimate and the *t*-statistic for SPOCQD, whereas the next two columns are the coefficient estimate and the *t*-statistic for the alternative predictor. The reported coefficients are scaled such that they measure the percentage change in annualized expected returns due to a one-standard-deviation increase. The *t*-statistics are based on Hodrick (1992) standard errors with lag length equal to the number of months in each horizon. The asterisks indicate statistical significance as follows: 1%(***), 5%(**), and 10%(*). The results are based on 268 monthly observations from January 1990 to April 2012. See Section 3.3.1 in Rapach and Zhou (2013) for a discussion of the series and associated mnemonics.

Table A7. Univariate regressions OOS economic performance: expanding window, ($\gamma = 2$)

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQ	0.42	0.32	0.28	0.16	SPOCQ	10.61	13.51	13.8	18.27
CAY	0.09	0.03	0.06	−0.04	CAY	0.17	0.12	0.07	0
DFSP	0.2	0.04	−0.02	−0.02	DFSP	3.56	5.9	5.12	6.55
Log(DY)	0.01	0.05	0.11	0.11	Log(DY)	3.79	4.55	4.68	6.03
Log(PE)	−0.07	−0.18	−0.18	−0.15	Log(PE)	2.12	2.59	2.77	3.69
RREL	0.25	−0.00	−0.10	−0.15	RREL	1.97	2.66	2.53	3.14
TBLL	−0.16	−0.21	−0.09	0.02	TBLL	2.13	2.1	2.12	1.74
TMSP	−0.18	0.01	0.19	0.33	TMSP	1.82	4.45	5.86	8.11
IV	0.03	−0.00	−0.04	−0.08	IV	6.73	8.5	7.68	8.71
RV	−0.14	0.02	−0.05	−0.12	RV	5.33	9.87	9.68	10.83
VRP	0.2	−0.03	−0.05	−0.04	VRP	22.11	17.73	16.14	16.38
MIN	−0.18	−0.21	−0.18	−0.15	MIN	0.17	0.12	0.07	0
MAX	0.25	0.05	0.19	0.33	MAX	22.11	17.73	16.14	16.38
(a) Sharpe ratio					(b) Relative portfolio turnover (RPT)				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQ	0.054	0.067	0.069	0.048	SPOCQ	7.5	6.49	5.06	3.18
CAY	0.024	0.03	0.034	0.012	CAY	3.83	3.98	3.19	2.37
DFSP	0.034	0.031	0.022	0.016	DFSP	4.8	3.47	1.84	1.28
Log(DY)	0.016	0.033	0.041	0.04	Log(DY)	0.97	2.67	3.14	2.37
Log(PE)	0.009	0.003	−0.005	−0.007	Log(PE)	−0.47	−2.08	−2.33	−2.68
RREL	0.038	0.026	0.008	−0.006	RREL	5.12	3.66	1.61	−0.09
TBLL	0.001	−0.000	0.01	0.023	TBLL	2.02	0.77	0.63	1.07
TMSP	−0.000	0.027	0.055	0.077	TMSP	0.01	2.53	4.4	5.21
IV	0.019	0.025	0.018	0.005	IV	2.02	1.64	0.85	−0.53
RV	0.003	0.028	0.016	−0.001	RV	1.07	3.14	1.29	−0.44
VRP	0.033	0.022	0.017	0.012	VRP	4.67	1.89	1.37	0.42
MIN	−0.000	−0.000	−0.005	−0.007	MIN	−0.47	−2.08	−2.33	−2.68
MAX	0.038	0.033	0.055	0.077	MAX	5.12	3.98	4.4	5.21
(c) Return loss (RL)					(d) Certainty equivalent return (CER)				

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a hypothetical investor with mean–variance utility function and coefficient of relative risk aversion γ equal to 2. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.

Table A8. Univariate regressions OOS economic performance: rolling window, ($\gamma = 2$)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	0.38	0.32	0.29	0.17
CAY	0.09	0.12	0.11	−0.04
DFSP	0.16	0.05	−0.02	−0.02
Log(DY)	0.03	0.05	0.11	0.12
Log(PE)	−0.06	−0.16	−0.17	−0.13
RREL	0.23	−0.02	−0.13	−0.16
TBLL	−0.14	−0.19	−0.06	0.04
TMSP	−0.22	0.03	0.21	0.33
IV	−0.02	0	−0.04	−0.09
RV	−0.13	0.02	−0.06	−0.12
VRP	0.15	−0.05	−0.06	−0.04
MIN	−0.22	−0.19	−0.17	−0.16
MAX	0.23	0.12	0.21	0.33

(a) Sharpe ratio

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	9.49	11.66	12.95	17.18
CAY	0.2	0.17	0.17	0
DFSP	2.42	5.18	4.98	6.34
Log(DY)	2.63	3.74	3.66	4.66
Log(PE)	1.89	2.18	2.69	3.48
RREL	1.73	2.2	2.34	2.68
TBLL	2	2	2.02	1.81
TMSP	2.05	3.74	5.03	6.84
IV	5.6	7.77	8.03	8.91
RV	5.55	8.81	9.32	10.62
VRP	17.63	15.02	14.35	15.61
MIN	0.2	0.17	0.17	0
MAX	17.63	15.02	14.35	15.61

(b) Relative portfolio turnover (RPT)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	0.05	0.066	0.07	0.051
CAY	0.024	0.041	0.041	0.013
DFSP	0.031	0.032	0.021	0.016
Log(DY)	0.019	0.032	0.042	0.042
Log(PE)	0.011	0.005	−0.002	−0.003
RREL	0.037	0.023	0.004	−0.009
TBLL	0.004	0.002	0.015	0.027
TMSP	−0.004	0.029	0.057	0.079
IV	0.014	0.026	0.017	0.004
RV	0.004	0.028	0.015	−0.001
VRP	0.028	0.02	0.014	0.012
MIN	−0.004	0.002	−0.002	−0.009
MAX	0.037	0.041	0.057	0.079

(c) Return loss (RL)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	7.07	6.38	5.17	3.35
CAY	3.85	3.92	3.18	2.42
DFSP	4.52	3.56	1.9	1.34
Log(DY)	0.5	1.9	3.01	2.44
Log(PE)	−0.58	−2.38	−2.49	−2.73
RREL	4.97	3.49	1.41	−0.23
TBLL	2.38	0.83	0.84	1.21
TMSP	−0.19	2.63	4.58	5.34
IV	1.6	1.86	0.84	−0.51
RV	1.31	3.09	1.28	−0.45
VRP	3.85	1.64	1.18	0.45
MIN	−0.58	−2.38	−2.49	−2.73
MAX	4.97	3.92	4.58	5.34

(d) Certainty equivalent return (CER)

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a a hypothetical investor with mean-variance utility function and coefficient of relative risk aversion γ equal to 2. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.

Table A9. Univariate regressions OOS economic performance: expanding window, ($\gamma = 10$)

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQ	0.46	0.35	0.28	0.19	SPOCQ	19.86	23.9	29.69	33.01
CAY	0.09	−0.02	−0.02	−0.04	CAY	0.12	0.09	0.06	0
DFSP	0.17	0.05	−0.01	−0.02	DFSP	3.04	5.55	5.49	6.53
Log(DY)	0.1	0.11	0.17	0.16	Log(DY)	4.08	5.05	6.62	8.77
Log(PE)	−0.12	−0.27	−0.24	−0.19	Log(PE)	4.95	5.74	6.56	8.25
RREL	0.26	−0.02	−0.13	−0.15	RREL	2.27	3.11	3.25	3.38
TBLL	−0.17	−0.21	−0.09	0.02	TBLL	1.89	1.99	2.14	1.97
TMSP	−0.18	0.01	0.21	0.35	TMSP	1.75	3.89	7.5	12.59
IV	0.04	0.03	−0.02	−0.09	IV	8.2	10.19	10.76	10.1
RV	−0.15	0	−0.07	−0.12	RV	4.41	9.58	10.79	11.13
VRP	0.29	−0.02	−0.04	−0.04	VRP	25.39	16.78	17.01	17.3
MIN	−0.18	−0.27	−0.24	−0.19	MIN	0.12	0.09	0.06	0
MAX	0.29	0.11	0.21	0.35	MAX	25.39	16.78	17.01	17.3
(a) Sharpe ratio					(b) Relative portfolio turnover (RPT)				

	Horizon (months)					Horizon (months)			
Predictor	6	12	18	24	Predictor	6	12	18	24
SPOCQ	0.012	0.014	0.014	0.011	SPOCQ	1.93	1.4	0.85	0.47
CAY	0.005	0.005	0.004	0.002	CAY	0.74	0.59	0.25	0.01
DFSP	0.007	0.007	0.004	0.003	DFSP	0.95	0.67	0.33	0.17
Log(DY)	0.005	0.008	0.01	0.01	Log(DY)	0.39	0.62	0.7	0.41
Log(PE)	0.001	−0.001	−0.003	−0.003	Log(PE)	−1.14	−1.29	−1.20	−0.96
RREL	0.008	0.005	0.001	−0.001	RREL	1.05	0.61	0.17	−0.07
TBLL	0	−0.000	0.002	0.005	TBLL	0.38	0.11	0.15	0.27
TMSP	0	0.006	0.011	0.016	TMSP	0	0.53	0.86	0.92
IV	0.004	0.006	0.004	0.001	IV	0.43	0.4	0.23	−0.05
RV	0.001	0.005	0.003	−0.000	RV	0.22	0.57	0.19	−0.09
VRP	0.009	0.005	0.003	0.002	VRP	1.2	0.38	0.25	0.09
MIN	0	−0.001	−0.003	−0.003	MIN	−1.14	−1.29	−1.20	−0.96
MAX	0.009	0.008	0.011	0.016	MAX	1.2	0.67	0.86	0.92
(c) Return loss (RL)					(d) Certainty equivalent return (CER)				

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a hypothetical investor with mean–variance utility function and coefficient of relative risk aversion γ equal to 10. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.

Table A10. Univariate regressions OOS economic performance: rolling window, ($\gamma = 10$)

(a) Sharpe ratio

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	0.44	0.35	0.28	0.21
CAY	0.09	0.04	0.02	−0.04
DFSP	0.14	0.05	−0.02	−0.02
Log(DY)	0.17	0.18	0.24	0.22
Log(PE)	−0.13	−0.27	−0.24	−0.18
RREL	0.23	−0.04	−0.15	−0.17
TBLL	−0.17	−0.21	−0.06	0.04
TMSP	−0.24	0.02	0.25	0.37
IV	−0.02	0.03	−0.04	−0.09
RV	−0.14	−0.01	−0.08	−0.12
VRP	0.24	−0.04	−0.06	−0.04
MIN	−0.24	−0.27	−0.24	−0.18
MAX	0.24	0.18	0.25	0.37

(b) Relative portfolio turnover (RPT)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	16.39	22.74	31.26	36.72
CAY	0.12	0.12	0.16	0
DFSP	2.09	4.52	4.94	6.33
Log(DY)	4.71	6.65	9.66	13.48
Log(PE)	4.48	5.6	6.67	8.62
RREL	1.77	2.57	2.95	2.92
TBLL	1.96	2	2.33	2.16
TMSP	2.05	3.73	7.82	13.23
IV	5.07	8.74	10.18	10.62
RV	4.08	8.22	10.08	10.94
VRP	17.1	13.42	14.42	16.47
MIN	0.12	0.12	0.16	0
MAX	17.1	13.42	14.42	16.47

(c) Return loss (RL)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	0.012	0.014	0.014	0.011
CAY	0.005	0.006	0.006	0.003
DFSP	0.006	0.007	0.004	0.003
Log(DY)	0.007	0.01	0.013	0.012
Log(PE)	0.001	−0.001	−0.003	−0.003
RREL	0.008	0.004	0	−0.002
TBLL	0.001	0	0.003	0.005
TMSP	−0.001	0.006	0.013	0.017
IV	0.003	0.006	0.004	0.001
RV	0.001	0.005	0.003	−0.000
VRP	0.008	0.004	0.003	0.002
MIN	−0.001	−0.001	−0.003	−0.003
MAX	0.008	0.01	0.013	0.017

(d) Certainty equivalent return (CER)

	Horizon (months)			
Predictor	6	12	18	24
SPOCQ	1.89	1.39	0.85	0.47
CAY	0.77	0.59	0.25	0.01
DFSP	0.92	0.69	0.33	0.18
Log(DY)	0.2	0.53	0.83	0.17
Log(PE)	−1.78	−1.76	−1.50	−1.16
RREL	1.03	0.57	0.12	−0.10
TBLL	0.42	0.1	0.2	0.32
TMSP	−0.09	0.57	1	1.06
IV	0.36	0.44	0.21	−0.05
RV	0.27	0.55	0.17	−0.09
VRP	1.04	0.35	0.22	0.1
MIN	−1.78	−1.76	−1.50	−1.16
MAX	1.04	0.69	1	1.06

Notes: We assess the OOS performance of the models in Table 3 against the HA by forecasting horizon in months in panels (b)–(d). In panel (a), we report the Sharpe ratio for each predictor. In all four panels, the statistics are calculated for a hypothetical investor with mean–variance utility function and coefficient of relative risk aversion γ equal to 10. The full sample is January 1990 to April 2012. The first OOS forecast date is September 2006. To ease the reader, the rows MIN and MAX show the minimum and maximum values of the criterion under consideration for predictors other than SPOCQD. For additional details, see Section 3.4.3 in the main text.