Import demand elasticities based on quantity data: Theory and evidence

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Abstract. Correct estimates of import demand elasticities are essential for measuring the gains from trade and predicting the impact of trade policies. We show that estimates of import demand elasticities hinge critically on whether they are derived using trade quantities or trade values, and this difference is due to properties of the estimators. Using partial identification methods, we show theoretically that the upper bound on the set of plausible estimates is lower when using traded quantities, compared to the standard approach using trade values. Our theoretical predictions are confirmed using detailed product-level data on US imports for the years 1993 to 2006. Our proposed method using traded quantities leads to smaller point estimates of the import demand elasticities for many goods and imply larger gains from trade compared to estimates based on trade values.

Résumé. Élasticités de la demande d'importation selon des données quantitatives: Théorie et preuves. De bonnes estimations des élasticités de la demande d'importation sont essentielles pour mesurer les gains du commerce et prévoir les répercussions des politiques commerciales. Nous montrons que ces estimations dépendent énormément de leur mode de calcul (à partir des quantités commerciales ou des valeurs commerciales) et que cette différence est attribuable aux propriétés des estimateurs. À l'aide de méthodes d'identification partielle, nous montrons qu'en principe, la limite supérieure de l'ensemble des estimations plausibles est plus faible lorsqu'on se sert des quantités commerciales que dans l'approche standard employant des valeurs commerciales. Nos prévisions théoriques sont confirmées par des données détaillées de produits relatives

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aux importations américaines des années de 1993 à 2006. La méthode que nous proposons employant les quantités commerciales mène à des estimations ponctuelles moindres des élasticités de la demande d'importation pour nombre de biens et laisse entendre de plus grands gains du commerce que les estimations fondées sur les valeurs commerciales.

JEL classification: F10, F12, F14, C52

1. Introduction

Correct estimates of the elasticity of import demand are crucial to accurately estimate the gains from trade, predict the impact of trade policies and impute the size of trade costs from data on international trade flows. The lower these estimates, the greater the benefits of international trade and economic integration in most trade models.

Estimations of the elasticity of import demand are traditionally performed using trade value data and "trade unit values" that are constructed by dividing trade values by trade quantities. An alternative approach is to estimate import demand elasticities using data on traded quantities instead of trade values. However, the international economics literature has avoided using import quantity data when estimating import demand elasticities, and authors typically claim that measurement error in the quantity data is an issue. The literature often cites Kemp (1962), who warned of the bias caused by measurement errors when estimating import demand elasticities.

The purpose of this paper is to show that the choice between trade values and traded quantity data for import demand elasticity estimations is not innocuous. We apply the method of partial identification of demand and supply elasticities developed by Leamer (1981) to estimate the upper and lower bounds on the set of possible estimates for the elasticity of import demand. Using detailed product-level data on US imports for the years 1993 to 2006, we estimate elasticities based on trade value versus trade quantity data. We show that using trade quantities yields estimates of import demand elasticity upper bounds that are substantially *smaller* than if trade values are employed. Using values instead of quantities tends to bias the upper bound estimates upwards. The variance of the relevant variable—quantity or value—is an important part of the upper bound, and because value has a higher variance than quantity, it tends also to have a higher upper bound. Because the lower bounds are identical using both approaches, this implies that *the range of plausible estimates is much smaller* when using traded quantities compared to the standard approach of using trade values. The pattern of the upper and lower bounds in both approaches closely matches our theoretical predictions for the asymptotic bias of each bound.

Given earlier authors' concerns regarding measurement error, we also theoretically derive the asymptotic bias of our estimators for the upper and lower bounds in the presence of measurement error in both trade quantities and trade values. We show that our original theoretical results are not overturned unless measurement error is sufficiently more severe in the quantity data than in the value data.

Import demand elasticity point estimates, typically derived using the the Feenstra (1994) methodology, sometimes have large confidence intervals (Ossa 2015). We argue that estimating upper and lower bounds on these elasticities is an alternative approach to capturing the uncertainty of these estimates. We adapt Feenstra's (1994) approach in order to derive point estimates based on traded quantity data and find that the point estimates based on quantity data are lower on average than the corresponding point estimates using trade value data.

Our results contribute to a recent literature that attempts to quantity the gains from trade for different countries and time periods employing workhorse models of international trade. Using the framework developed by Arkolakis et al. (2012) and Ossa (2015), we show that the demand elasticity point estimates using traded quantity data imply larger gains from trade compared to the traditional approach using point estimates based on trade value data. We argue that the quantity-based point estimates or the quantity-based upper bounds of the demand elasticities provide an alternative to using value-based point estimates to gauge the gains from trade.

Our results also have important implications for previous studies that measure various impacts of trade using import demand elasticities based on trade values. Prominent examples include previous studies of the gains from increased variety due to imports (Broda and Weinstein 2006) and the size of trade costs (Jacks et al. 2008, Jacks et al. 2011, Chen and Novy 2011, Novy 2013). Import demand elasticities have also been used in the calibration of countless applied models of international trade. Import demand elasticities are commonly used to calculate the trade elasticity, and this approach is conceptually distinct from estimates of the trade elasticity using international price differences (Eaton and Kortum 2002, Simonovska and Waugh 2014), tariff fluctuations (Hummels 1999, Baier and Bergstrand 2001, Head and Ries 2001, Romalis 2007, Caliendo and Parro 2015, Berthou and Fontagne 2016, Bas et al. 2017, Fontagné et al. 2019) and exchange rate fluctuations (Berman et al. 2012, Fitzgerald and Haller 2018).

While we test and motivate our analysis in the context of international trade, our results are generalizable to any estimation of demand elasticities where price data must be constructed from quantity and value data, and the econometrician must select the most appropriate model. For example, household survey data on expenditures and quantities are used to estimate price elasticities (Deaton 1987, Deaton 1990). Unit values are also prevalent in firm-level data sets and are used to estimate price elasticities for unit labour costs (Carlsson and Skans 2012) and electricity unit values (Davis et al. 2013).

The rest of the study proceeds as follows. In section 2, we present the theory behind the partial identification of the import demand elasticities and derive the asymptotic bias associated with the upper and lower bound estimators. Section 3 describes our data and empirical methodology, including how we derive point estimates based on traded quantity data. In section 4, we present the results estimating the upper bounds, lower bounds and point estimates of the import demand elasticity using US import data. Given our new estimates, we quantify the impact of these new estimates on the welfare gains from trade in section 5. Section 6 concludes.

2. Partially identifying import demand elasticities

We begin by theoretically deriving the difference in asymptotic bias when estimating import demand elasticities using quantity data or value data. The import demand elasticity for a good can be naively estimated by regressing traded quantities on prices:

$$\ln x_{ct} = -\beta \ln p_{ct} + \varepsilon_{ct}, \tag{1}$$

where x_{ct} is the quantity demanded from country c in year t and p_{ct} is its corresponding price.[1] However, estimating equation (1) by OLS will lead to biased and inconsistent estimates of β if the errors are correlated with prices, i.e., $E(\varepsilon_{ct} \ln p_{ct}) > 0$. This positive covariance arises if ε_{ct} contains demand shocks—a positive demand shock raises both quantity and price. An IV approach is one potential solution, but the absence of good instruments in this context has lead to alternative approaches in the literature.

The challenge of estimating import demand and supply elasticities in the absence of good instruments has a long tradition in economics. The study of an under-identified supply and demand system was pioneered by Working (1927), who shows that under certain conditions the data trace out the demand curve if the supply curve is more variable than the demand curve. Leamer (1981) shows that in a demand−supply system with zero covariance between the residuals, the set of possible maximum likelihood estimates is defined by a hyperbola.[2] Leamer (1981) also shows that if the demand elasticity is assumed to be negative and the supply elasticity is assumed to be positive, then the set of maximum likelihood estimates for one elasticity is the interval between the direct least-squares estimate (regressing quantities on prices) and the reverse least-squares estimates (regressing prices on quantities). Leamer (1981) furthermore establishes that equation (1) defines either the upper or the lower bound on the true estimate of the demand elasticity and that the reverse least square estimate will define the other bound. In

1 Note that we express the elasticity of demand as a positive value. For simplicity and without loss of generality, we omit the constant in the regression equation and assume all variables have mean zero.

2 Leamer shows this result for a time series on a single good, whereas we work with a cross-country panel. However, there is nothing specific to the nature of the variation that determines the result. Given the specification in equation (1) and the Leamer assumptions, the result holds whether variation arises over time or across countries.

what follows, we employ Leamer's (1981) partial identification approach to estimating an upper and a lower bound for the elasticity of import demand.

2.1. Quantity–price approach (Leamer 1981)

The main principle of partial identification is to estimate an interval in which the true parameter lies. Establishing a valid interval requires proving that the upper bound is above the true parameter and the lower bound is below the true parameter. For these bounds to be informative, the interval should be as narrow as possible, while at the same time ensuring that the bounds bracket the parameter of interest.

We now derive the asymptotic bias of the estimators for the least squares and reverse least squares regressions of import quantities on import prices. The demand equation is given by equation (1), and the supply equation is given by

$$\ln x_{ct} = \gamma \ln p_{ct} + \eta_{ct}, \tag{2}$$

which yields the following reduced form:

$$\ln x_{ct} = \frac{\gamma}{\gamma+\beta}\varepsilon_{ct} + \frac{\beta}{\gamma+\beta}\eta_{ct}$$
$$\ln p_{ct} = \frac{1}{\gamma+\beta}\varepsilon_{ct} - \frac{1}{\gamma+\beta}\eta_{ct}$$

The probability limit of the OLS estimate of β using (1) is[3]

$$\texttt{plim } \hat{\beta} = -\frac{E(\ln x_{ct}\ln p_{ct})}{E\left((\ln p_{ct})^2\right)} = \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\gamma-\beta)\sigma_{\varepsilon\eta}}{\sigma_\varepsilon^2 + \sigma_\eta^2 - 2\sigma_{\varepsilon\eta}}, \tag{3}$$

where $\sigma_{\varepsilon\eta} = E(\varepsilon_{ct}\eta_{ct})$, $\sigma_\varepsilon^2 = var(\varepsilon_{ct})$ and $\sigma_\eta^2 = var(\eta_{ct})$. Now, consider the reverse regression of $\ln p_{ct}$ on $\ln x_{ct}$. The probability limit of the OLS estimator is

$$\texttt{plim } \hat{\beta}^R = -\frac{E(\ln x_{ct}\ln p_{ct})}{E\left((\ln x_{ct})^2\right)} = \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\gamma-\beta)\sigma_{\varepsilon\eta}}{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\beta+\gamma)\sigma_{\varepsilon\eta}}. \tag{4}$$

Assume the supply and demand shocks are uncorrelated, i.e., $\sigma_{\varepsilon\eta} = 0$. One can then see that $\texttt{plim } \hat{\beta}$ is a weighted average of β and γ. It can also be shown that the inverse of $\texttt{plim } \hat{\beta}^R$ lies between β^{-1} and γ^{-1}.

We can express the probability limits for the least squares and reverse least squares estimates to illustrate how much they differ from the true β:[4]

3 Throughout, we assume that the data satisfy sufficient moment and dependence conditions for a law of large numbers to hold.

4 Leamer (1981) shows that the hyperbola of the maximum likelihood estimates is given by $\hat{\gamma}^2\left(\hat{\beta}s_p^2 - s_{px}\right) + \hat{\beta}^2\left(-\hat{\gamma}s_p^2 + s_{px}\right) = \left(\hat{\beta} - \hat{\gamma}\right)s_x^2$, where s_p^2 and s_x^2 are the sample variances and s_{px} is the sample covariance. Assuming a

$$\texttt{plim}\ \hat{\beta} = \beta - (\gamma + \beta)\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \leq \beta \tag{5}$$

$$\frac{1}{\texttt{plim}\ \hat{\beta}^R} = \beta + (\gamma + \beta)\frac{\gamma\sigma_\varepsilon^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2} \lesseqgtr \beta \tag{6}$$

It is clear from equation (5) that the least squares estimate, which captures the lower bound, brackets the true β from below. With an additional parametric assumption on the sign of the denominator in equation (6) implicitly made by Leamer (1981), we obtain the Leamer result that the least squares and reverse least squares estimates constitute the upper and lower bound on β:

$$0 \leq \texttt{plim}\ \hat{\beta} \leq \beta \leq \frac{1}{\texttt{plim}\ \hat{\beta}^R} \Leftrightarrow \beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 > 0 \tag{7}$$

The parametric assumption in the denominator of equation (7) stems from the fact that we require more variation in the supply equation than the demand equation in order to trace out the demand curve, which was pointed out by Working (1927).

2.2. Value-based approach

In international trade data, the price is constructed as the average unit value of each trade flow, i.e., $p_{ct} = v_{ct}/x_{ct}$, where v_{ct} is the value of trade. Taking logs and rearranging yields

$$\ln v_{ct} = \ln p_{ct} + \ln x_{ct}. \tag{8}$$

This simple relationship between trade values, trade quantities and trade unit values in the data implies that β and γ can be estimated using any two of the components from equation (8) and then transforming the resulting point estimate. For example, one can use equation (8) to transform equations (1) and (2) into regression of trade values on trade unit values, yielding the following expressions for demand and supply:

$$\ln v_{ct} = (1 - \beta) \ln p_{ct} + \varepsilon_{ct} \tag{9}$$

$$\ln v_{ct} = (\gamma + 1) \ln p_{ct} + \eta_{ct} \tag{10}$$

The reduced form of this system of equations is given by

$$\ln v_{ct} = \frac{1+\gamma}{\gamma+\beta}\varepsilon_{ct} + \frac{\beta-1}{\gamma+\beta}\eta_{ct},$$

$$\ln p_{ct} = \frac{1}{\gamma+\beta}\varepsilon_{ct} - \frac{1}{\gamma+\beta}\eta_{ct}.$$

non-negative supply elasticity, the upper bound for the demand elasticity is found by imposing $\hat{\gamma} = 0$, which yields $\hat{\beta} = \frac{s_x^2}{s_{px}}$, the inverse of the least squares estimate of p on x.

To define a lower bound on β, regress $\ln v_{ct}$ on $\ln p_{ct}$. Define the coefficient in this regression to be $\theta = 1 - \beta$. The probability limit of the estimated lower bound on β is therefore

$$1 - \texttt{plim}\ \hat{\theta} = 1 - \frac{E(\ln v_{ct} \ln p_{ct})}{E\left((\ln p_{ct})^2\right)} = \beta - (\gamma + \beta)\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \leq \beta. \tag{11}$$

Note that the probability limit of the lower bound in equation (11) is *identical* to equation (5). This stems from the fact that price is on the right-hand side when estimating the lower bound, regardless of whether quantities or values are the dependent variable.

The upper bound, however, is not identical to the quantity−price approach. Define θ^R as the coefficient in a reverse regression of ln p_{ct} on ln v_{ct}. The probability limit for the upper bound of β based on this regression takes the following form:[5]

$$\begin{aligned}\texttt{plim}\ \left(-\frac{1-\hat{\theta}^R}{\hat{\theta}^R}\right) &= -\frac{1 - \frac{E(\ln v_{ct} \ln p_{ct})}{E((\ln v_{ct})^2)}}{\frac{E(\ln v_{ct} \ln p_{ct})}{E((\ln v_{ct})^2)}} \\ &= \beta + (\gamma + \beta)\frac{(1+\gamma)\sigma_\varepsilon^2}{(\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2) - (\sigma_\eta^2 + \sigma_\varepsilon^2)}\end{aligned} \tag{12}$$

The denominator of the last term in equation (12) is smaller than the corresponding term in equation (6) and the numerator is larger. This means that, if the denominator is positive (i.e., $\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 > \sigma_\eta^2 + \sigma_\varepsilon^2$), then the value-based upper bound exceeds the quantity-based upper bound. Moreover, if the denominator is negative, then equation (12) is no longer an upper bound on β. In short, equation (12) is either a less informative upper bound than equation (6), or is not an upper bound for β.

The bounds in equations (6) and (12) are most informative when they exceed but are close to β, i.e., when β is large relative to γ and/or σ_η^2 is large relative to σ_ε^2. It is common in the literature to assume $\beta > 1$. Feenstra (1994) assumes a demand elasticity in excess of unity due to CES preferences, and Scobie and Johnson (1975) argue that the elasticity of demand will be elastic if supplying countries are sufficiently "small" in the sense that there are several suppliers of a similar good to the export market.[6] If the variance of the supply shocks (σ_η^2) is large relative to variance of the demand

5 Inverting equation (9) without the error term yields the transformed reverse least squares coefficient $\theta^R = \frac{1}{1-\hat{\beta}}$. Rearranging yields $\beta = -\frac{1-\hat{\theta}^R}{\hat{\theta}^R}$.

6 As suggested by Scobie and Johnson (1975), another way to partially estimate import demand elasticities is to regress $\ln x_{ct}$ on $\ln v_{ct}$ and vice versa, thus avoiding the need to construct price data. We derive the asymptotic bias of the upper and lower bounds using this approach in the appendix. We find that the

shocks (σ_ε^2), then it means that more of the variation in the data comes from shifts of the supply curve along the demand curve rather than shifts of the demand curve, which in turn enables more precise identification of the demand elasticity. In the Feenstra (1994) model, low σ_ε^2 implies little variation in taste parameters or in the number of new or disappearing products.

To illustrate the relationships between equations (6) and (12), we plot the predicted asymptotic biases of each estimator for various values of the true import demand elasticity. The results of this exercise are reported in figure 1, where we plot β between 0 and 10 and hold constant $\gamma = 1$, $\sigma_\varepsilon^2 = 0.5$ and $\sigma_\eta^2 = 1.0$.[7] It is evident from the figure that the upper bound based on trade value data is *larger* than the Leamer upper bound for most values of β. Figure 1 also illustrates that the upper bound based on trade value data is highly unstable at low values of β and becomes negative when the true import demand elasticity is sufficiently low to make the denominator in equation (12) negative. The quantity-based approach is thus particularly well suited to situations where the true import demand elasticity is low.[8]

2.3. Measurement error

Next, we investigate whether our theoretical results hold in the presence of measurement error. Kemp (1962) was the first to warn of the bias caused by measurement errors when using quantity data for the purpose of estimating import demand elasticities. In Kemp's case, the bias was caused by

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quantity–value lower bound is identical to the Leamer upper bound and that the quantity–value upper bound is identical to the upper bound based on trade value data. Because the lower bound is not likely to bracket the true elasticity in this case, estimating import demand elasticities without constructing trade unit values thus leads to implausibly high estimates.

7 We assume a unit elastic export supply in accordance with average estimates from the literature and our own analysis. Tokarick (2014) estimates export supply elasticities for 87 countries and finds a median of 0.39 to 0.62 and mean of 0.45 to 0.83. Broda et al. (2008) finds a median of 1.1 and a mean of 3.6 for the US. In our analysis, we find a median of 0.25 to 0.29 and a mean of 2.4 to 3.2 using traded quantity data, depending on the level of product aggregation. These estimates are reported in the appendix.

8 The problem of invalid upper bounds using value data compared with quantity data becomes worse when increasing the assumed ratio of error variances, σ_ε^2 and σ_η^2. If we instead assume $\sigma_\varepsilon^2 = \sigma_\eta^2 = 1$, then the value-based upper bound estimates are invalid for $\beta < 3.0$ while the Leamer upper bound estimates are invalid only for $\beta < 1.0$. Increasing both error variances in proportion increases both the Leamer bounds and value-based bounds, but does not qualitatively change the results.

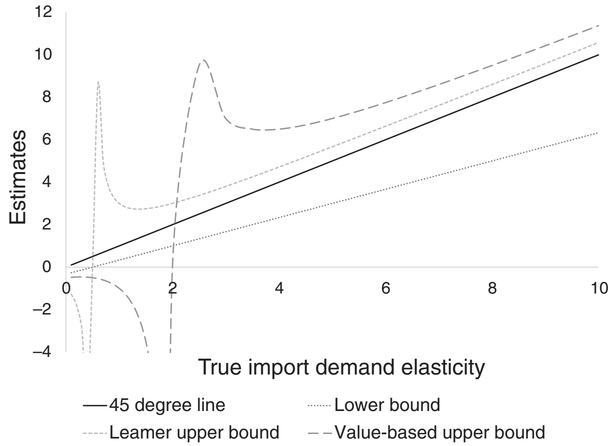

FIGURE 1 Theoretically predicted upper and lower bounds as function of true import demand elasticity, no measurement error
NOTE: $\gamma = 1, \sigma_{\varepsilon}^2/\sigma_{\eta}^2 = 0.5$ in all cases.
SOURCE: Authors' calculations

constructing quantity indices from trade value and price index data. In the second paragraph of Kemp (1962), he writes:

> In studies using product-level data, however, the quantity variables almost always is constructed by dividing the index of import prices into an index of the total money value of imports. The quantity variable is subject therefore to a measurement error of its own.

In his derivations, Kemp assumes a measurement error term in the price index data, but not in the money value of imports. Kemp goes on to show that using constructed quantity index data leads to biased and inconsistent estimates of the import demand elasticity, which correspond to our lower bound estimates. In the context of contemporary international trade data, however, the raw Comtrade data report the value of trade and its quantity (in weight or units). We thus argue that Kemp's case for error in the quantity data is no longer relevant. However, there are many aspects to consider when comparing the relative size of measurement error in quantities and values, which we now discuss.

Both trade values and traded quantity data may be subject to measurement error for various reasons, but there is a dearth of studies that compare the relative magnitude of measurement error in these two types of trade data. Many of the sources of measurement error in trade data afflict both traded value and traded quantity data, such as incorrectly reported product codes, re-export trans-shipments, data entry mistakes and transfer pricing. However, there are some sources of measurement error sources that are specific to just trade quantity data or just trade value data. Quantity units might be heterogeneous for specific products, and it might be difficult to ascertain quantity measures for some products.[9] There is an incentive to under-report traded quantities if tariffs are per unit, but most import tariffs are ad valorem, implying a greater risk under-reporting import trade values or evading value-added taxes by under-reporting export values. Measurement error stemming from exchange rate fluctuations is a problem specific to trade values and can occur if the customs authority must convert from a foreign currency to a domestic currency when reporting trade data. Goldberg and Tille (2008) find, for example, that foreign currencies feature more prominently in the invoicing of US imports from the EU, the UK and Japan.

To allow for measurement error, we express the observed data as

$$\begin{aligned} \ln v_{ct} &= \ln \tilde{v}_{ct} + u_{ct} \\ \ln x_{ct} &= \ln \tilde{x}_{ct} + w_{ct}, \end{aligned}$$

where $\tilde{v}_{ct}$ and $\tilde{x}_{ct}$ denote the true (unobserved) data. The measurement error variances and covariances are σ_u^2, σ_w^2 and σ_{uw}. Because $\ln p_{ct} = \ln v_{ct} - \ln x_{ct}$, the measurement error in prices is $u_{ct} - w_{ct}$. We assume classical measurement error, i.e., the measurement errors are uncorrelated with the true values.[10]

We first present the probability limits on the bounds in the quantity-based specification. Incorporating measurement error, the probability limit of β using equation (1) is

$$\begin{aligned} \texttt{plim}\ \hat{\beta} &= -\frac{E(\ln x_{ct} \ln p_{ct})}{E\left((\ln p_{ct})^2\right)} \\ &= \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\beta+\gamma)^2\left(\sigma_w^2 - \sigma_{uw}\right)}{\sigma_\varepsilon^2 + \sigma_\eta^2 + (\beta+\gamma)^2\left(\sigma_u^2 + \sigma_w^2 - 2\sigma_{uw}\right)} \\ &= \beta - (\beta+\gamma)\frac{\sigma_\varepsilon^2 + \beta(\beta+\gamma)(\sigma_u^2 - \sigma_{uw}) + (\beta-1)(\beta+\gamma)(\sigma_w^2 - \sigma_{uw})}{\sigma_\varepsilon^2 + \sigma_\eta^2 + (\beta+\gamma)^2\left(\sigma_u^2 + \sigma_w^2 - 2\sigma_{uw}\right)}. \end{aligned} \tag{13}$$

9 There generally tend to be more observations with missing quantity data, while the invoiced value is always reported. This is driven by that fact that customs authorities require an invoiced value to levy ad valorem import tariffs, which are most common. This issue of missing data is separate from the issue of measurement error.

10 Unobserved quality can also be treated as a component of the measurement error. For example, define the $\ln \tilde{v}_{ct}$ to be the value of country c's product if it were of average quality and u_{ct} to be the quality differential. This term would also appear in the price because prices are constructed from unit values, i.e., $p_{ct} = v_{ct}/x_{ct}$.

For the reverse regression, we have

$$\begin{aligned}
\frac{1}{\texttt{plim } \hat{\beta}^R} &= -\frac{E\left((\ln x_{ct})^2\right)}{E(\ln x_{ct} \ln p_{ct})} \\
&= \frac{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\beta+\gamma)^2\sigma_w^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_{uw} - \sigma_w^2\right)} \\
&= \beta + (\beta+\gamma)\frac{\gamma\sigma_\varepsilon^2 + \beta(\beta+\gamma)\left(\sigma_{uw} - \sigma_w^2\right) + (\beta+\gamma)\sigma_w^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_{uw} - \sigma_w^2\right)}. \qquad (14)
\end{aligned}$$

For the value-based approach, the probability limit of $\hat{\theta}$ using the direct regression in equation (9) is identical to equation (13) in the presence of measurement error. As in equation (12), the reverse regression is not identical to the quantity-based upper bound, and it takes the following form:

$$\begin{aligned}
&\texttt{plim } \left(-\frac{1-\hat{\theta}^R}{\hat{\theta}^R}\right) \\
&= \frac{\gamma(1+\gamma)\sigma_\varepsilon^2 + \beta(\beta-1)\sigma_\eta^2 + (\beta+\gamma)^2\sigma_u^2 - (\beta+\gamma)^2\left(\sigma_u^2 - \sigma_{uw}\right)}{(\beta-1)\sigma_\eta^2 - (1+\gamma)\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_u^2 - \sigma_{uw}\right)} \\
&= \beta + (\beta+\gamma)\frac{(1+\gamma)\sigma_\varepsilon^2 + (\beta+\gamma)\sigma_u^2 + (\beta-1)(\beta+\gamma)\left(\sigma_u^2 - \sigma_{uw}\right)}{(\beta-1)\sigma_\eta^2 - (1+\gamma)\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_u^2 - \sigma_{uw}\right)} \qquad (15)
\end{aligned}$$

The parameter restrictions required for the bounds to hold in the presence of measurement error are now more complicated because they also hinge on the magnitudes of the error variance and covariance. We therefore study three specific cases of measurement error. In the first case, we assume that the measurement error variance in traded quantities and trade values, and their covariance, are equal in magnitude, which we call the "quantity and value error" case. This case implies that there is no error in the unit values (prices) on average. In the second and third cases, we assume that there is measurement error in either traded quantities or trade values. The results of this exercise are illustrated in figure 2. In all cases, when a measurement error is non-zero, we set its variance equal to 5% of the variance of the supply shock (σ_η^2).

For the lower bound, equal quantity and value error causes the measurement error to drop out of equation (13), so the quantity and value error case is identical to no measurement error. If $\beta > 1$, then both quantity and value measurement error reduce the lower bound, so the bound remains valid for any parameter values. For $\beta < 1$, however, value measurement error increases the bound and it may become invalid depending on the other parameters. The top panel of figure 2 shows that the lower bound becomes uninformative for large values of β, especially for value measurement error.

For the upper bound, measurement error in quantities only attenuates the quantity-based bound, and measurement error in values only attenuates

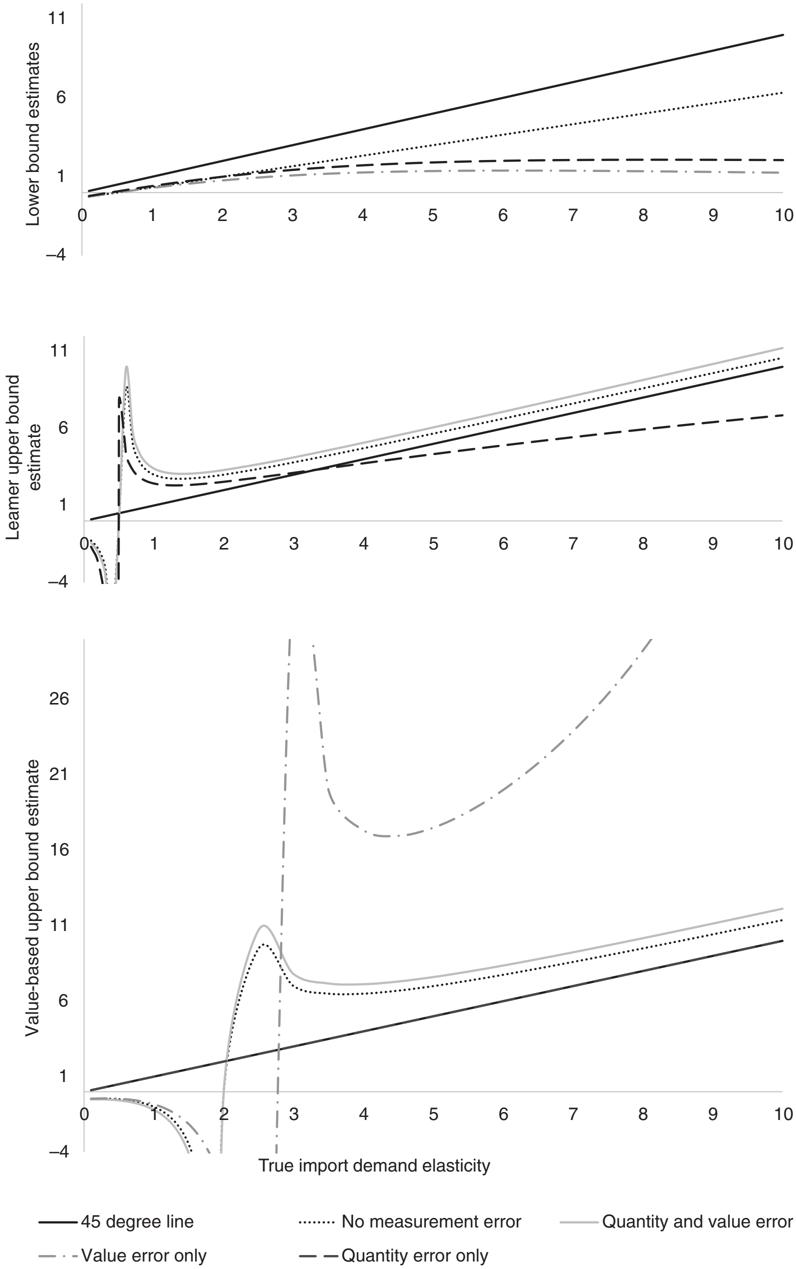

FIGURE 2 Theoretically predicted lower and upper bounds with measurement error
NOTES: $\gamma = 1, \sigma_\varepsilon^2/\sigma_\eta^2 = 0.5$ in all cases. $\sigma_u^2/\sigma_\eta^2 = 0, \sigma_w^2/\sigma_\eta^2 = 0.05$ in quantity measurement error case. $\sigma_u^2/\sigma_\eta^2 = 0.05, \sigma_w^2/\sigma_\eta^2 = 0$ in value measurement error case. $\sigma_u^2/\sigma_\eta^2 = \sigma_w^2/\sigma_\eta^2 = \sigma_{uw}/\sigma_\eta^2 = 0.05$ in quantity and value error case.
SOURCE: Authors' calculations

the value-based bound. The middle panel of figure 2 shows the true import demand elasticity along with the quantity-based upper bound with no measurement error, with measurement error of equal magnitude in quantities and values, and with measurement error in quantities only. Equal quantity and value error unambiguously increases the numerator in the bound formula. Thus, just as for the no measurement error case, the bound remains valid for any value of β above a threshold. Measurement error in quantities only attenuates the quantity-based upper bound, and the bound may be invalid for large values of β.

The bottom panel of figure 2 shows the true import demand elasticity along with the upper bound based on trade value data with no measurement error, with measurement error of equal magnitude in quantities and values, and with measurement error in values only. As for the quantity-based case, equal quantity and value error unambiguously increases the numerator in equation (14), and the bound is valid for any value of β above a threshold. Measurement error in values only inflates the upper bound based on trade value data and increases the threshold value of β at which it becomes invalid.

Overall, our partial identification theoretical results suggest that it is best to estimate import demand elasticities using traded quantities if there is relatively low measurement error in general, or if the error is similar in magnitude in the quantity and value data, i.e., there is little measurement error in prices. If measurement error in the data is suspected to be large, then the choice between using traded quantity data or trade value data becomes more pertinent. In this case, the econometrician should use the data with the least measurement error. In general, quantity data are particularly well suited to estimating import demand elasticities for goods with an expected low elasticity.[11] We summarize these theoretical predictions in table 1.

3. Exactly identifying import demand elasticities

We now generalize the model in equation (1) to allow for time- and country-specific effects. This brings our setting into alignment with Feenstra (1994), which enables us to apply his estimator. The model is

$$\ln x_{ct} = \psi_t + \alpha_c - \beta \ln p_{ct} + \varepsilon_{ct}. \tag{16}$$

To remove the time- and country-specific effects, we difference with respect to time and a reference country k to obtain

$$\Delta^k \ln x_{ct} = -\beta \Delta^k \ln p_{ct} + \Delta^k \varepsilon_{ct}, \tag{17}$$

11 Formally, it follows from equations (13), (14) and (15) that quantity data yield a valid upper bound estimate and value data yield an invalid upper bound estimate if $1/\texttt{plim}\ \hat{\beta}^R > \texttt{plim}\ \hat{\beta}$ and $1/\texttt{plim}\ \hat{\beta}^{RP} < \texttt{plim}\ \hat{\beta}$.

TABLE 1

When to use trade quantity data vs. trade value data in the presence of measurement error

Nature of measurement error	True value of demand elasticity	
	Low	High
Low in general: $\sigma_u^2 = \sigma_w^2 \approx 0$	Use quantity data	Use quantity data
Identical: $\sigma_u^2 = \sigma_w^2 = \sigma_{uw}$	Use quantity data	Use quantity data
Lower in quantities: $\sigma_u^2 > \sigma_w^2 > 0$	Use quantity data	Use quantity data
Lower in values: $\sigma_w^2 > \sigma_u^2 > 0$	Use quantity data	Use value data

NOTES: The definition of a low vs. a high value of the true demand elasticity follows from equations (13), (14) and (15). Formally, the value of β is defined as low if the following two conditions hold: $1/\texttt{plim}\ \hat{\beta}^R > \texttt{plim}\ \hat{\beta}$ and $1/\texttt{plim}\ \hat{\beta}^{RP} < \texttt{plim}\ \hat{\beta}$. The value of β is defined as high if $1/\texttt{plim}\ \hat{\beta}^{RP} > \texttt{plim}\ \hat{\beta}$.

where

$$\Delta^k \ln x_{ct} \equiv \Delta \ln x_{ct} - \Delta \ln x_{kt},$$

$$\Delta^k \ln p_{ct} \equiv \Delta \ln p_{ct} - \Delta \ln p_{kt},$$

$$\Delta^k \varepsilon_{ct} \equiv \Delta\varepsilon_{ct} - \Delta\varepsilon_{kt}.$$

Similarly, the supply equation is

$$\Delta^k \ln x_{ct} = \gamma \Delta^k \ln p_{ct} + \Delta^k \eta_{ct}. \tag{18}$$

This generalization does not change any of our results in section 2. For example, if we re-define $\hat{\beta}$ and $\hat{\beta}^R$ as OLS estimates computed using the double-differenced variables $\Delta^k \ln x_{ct}$ and $\Delta^k \ln p_{ct}$ and if we re-define σ_ε^2 and σ_η^2 to represent variances of the differenced error terms $\Delta^k \varepsilon_{ct}$ and $\Delta^k \eta_{ct}$, then the Leamer bounds expressions in section 2.1 remain unchanged.

Feenstra (1994) specifies a constant-elasticity-of-substitution model of input demand and derives the following estimating equation:[12]

$$\Delta \ln s_{ct} = \phi_t - (\beta - 1)\Delta \ln p_{ct} + \Delta\varepsilon_{ct}, \tag{19}$$

where $s_{ct} = v_{ct}/\sum_j v_{jt}$ denotes the cost share of country c. Under the Armington assumption, the elasticity of substitution β equals the import-demand elasticity. Differencing with respect to a reference country cancels the denominator in the cost share and the time effect ϕ_t, and it implies

$$\Delta^k \ln v_{ct} = -(\beta - 1)\Delta^k \ln p_{ct} + \Delta^k \varepsilon_{ct}. \tag{20}$$

12 This is equation (7) in Feenstra (1994), except we use β rather than σ for the elasticity of substitution and define the error term as $\Delta\varepsilon_{ct}$ rather than ε_{ct} to match our notation in equation (16).

Recalling that $\ln v_{ct} = \ln x_{ct} + \ln p_{ct}$, we can subtract $\Delta^k \ln p_{ct}$ from both sides to obtain the import demand equation in equation (17). Feenstra (1994) specifies a logarithmic supply equation as in equation (18).[13] Thus, the estimating equations derived from Feenstra's model are the same as ours.

In section 2, we derived bounds for β based on forward and reverse OLS regressions estimated using data pooled across countries and over time. Soderbery (2015) shows that the estimator proposed by Feenstra (1994) can be formulated from forward and reverse OLS regressions estimated separately by country. Assuming that the error variances in equations (17) and (18) differ across countries, estimates from two countries would be sufficient to uniquely identify the import demand elasticity. In general, Feenstra's method uses more than two countries and obtains a point estimate using a weighted least squares approach.

To develop an analog to Feenstra's method using data on traded quantities instead of trade values, multiply the error terms in equations (17) and (18) to obtain

$$\Delta^k \varepsilon_{ct} \Delta^k \eta_{ct} = \left(\Delta^k \ln x_{ct} + \beta \Delta^k \ln p_{ct}\right)\left(\Delta^k \ln x_{ct} - \gamma \Delta^k \ln p_{ct}\right). \tag{21}$$

Divide through by $\beta\gamma$, average over time, and rearrange to obtain

$$\begin{aligned} T^{-1}\sum_{t=1}^{T}\left(\Delta^k \ln p_{ct}\right)^2 &= \frac{1}{\beta\gamma} T^{-1}\sum_{t=1}^{T}\left(\Delta^k \ln x_{ct}\right)^2 \\ &\quad + \frac{\beta-\gamma}{\beta\gamma} T^{-1}\sum_{t=1}^{T}\left(\Delta^k \ln x_{ct} \Delta^k \ln p_{ct}\right) + u_c, \end{aligned} \tag{22}$$

where $u_c = T^{-1}\sum_{t=1}^{T}\left(\Delta^k \varepsilon_{ct} \Delta^k \eta_{ct}\right)/\beta\gamma$. Following Feenstra, we estimate equation (22) by weighted least squares and solve for β.[14]

To connect equation (22) to forward and reverse regressions, divide through by the covariance term $T^{-1}\sum_{t=1}^{T}\left(\Delta^k \ln x_{ct} \Delta^k \ln p_{ct}\right)$ cross product term to obtain

$$\frac{1}{\hat{\beta}_c} = \frac{1}{\beta\gamma}\frac{1}{\hat{\beta}_c^R} + \frac{\beta-\gamma}{\beta\gamma} + \hat{u}_c, \tag{23}$$

where $\hat{\beta}_c$ is the OLS estimate from the forward regression of $\Delta^k \ln x_{ct}$ on $\Delta^k \ln p_{ct}$ for country c, $\hat{\beta}_c^R$ is the OLS estimate from the reverse regression of $\Delta^k \ln p_{ct}$ on $\Delta^k \ln x_{ct}$ for country c, and $\hat{u}_c = u_c/T^{-1}\sum_{t=1}^{T}\left(\Delta^k \ln x_{ct} \Delta^k\right.$

13 He writes the supply equation as $\Delta \ln p_{ct} = \omega \Delta \ln x_{ct} + \xi_{ct}$, which is equivalent to equation (18) with $\gamma = 1/\omega$ and $\Delta^k \eta_{ct} = \omega(\xi_{ct} - \xi_{kt})$.

14 As noted previously, Feenstra's estimator uses values v_{ct} in place of quantities x_{ct}, so his coefficients are a different function of β.

$\ln p_{ct}$). The Feenstra estimator is equivalent to a weighted least squares regression of $1/\hat{\beta}_c$ on $1/\hat{\beta}_c^R$.

Feenstra (1994) shows that his estimator is consistent for β under the assumption that the error variances in equations (17) and (18) are not identical across countries. We implement the most recent refinement of Feenstra's method, by Soderbery (2015), who applies a limited information maximum likelihood (LIML) estimator to reduce bias and improve constrained search efficiencies.

4. Empirical analysis

4.1. Data

Our main data source is the US import data available at the Center for International Data, which is based on data from the United States Customs Service.[15] The data include the value of US imports (in USD) and its associated quantity by country of origin at the 10-digit Harmonized System (HS) level. We focus on the years 1993 to 2006. From the trade values and trade quantities, we compute trade unit values. We thus observe the trade value, trade quantity and trade unit values by HS product, partner country and year. We perform our estimations at the 8-, 6- and 4-digit HS levels, which we achieve by aggregating the data across products.

There are many advantages to using US data in our study. First, the US is a large importer that imports from many countries, which provides us with sufficient observations to estimate elasticities even within narrowly defined product categories. Second, customs data are generally regarded as more reliable when reported by the importer and reported by developed countries such as the US.[16] Third, the quantity observations reported in the US data are reported in their original units, which reduces measurement error stemming from harmonizing to a single type of quantity unit.[17] Finally, using US data allow us to relate our results to those of Feenstra (1994), Broda and Weinstein (2006) and Soderbery (2015).

There are 47 different types of quantity units in the raw trade data. We drop trade flow observations when the quantity unit is unreported. We also consolidate quantity unit abbreviations with identical descriptions. For example, we rewrite "HND" as "HUN" so as to have only one code for trade flows reported as "hundreds." This leaves us with 39 different quantity units in

15 See Feenstra et al. (2002) for a detailed description of the US import data. The data can be found at `http://cid.econ.ucdavis.edu/usix.html`.

16 Hummels and Lugovskyy (2006) posit that there is generally more measurement error in exporter-reported data because customs authorities in importing countries want accurate data in order to charge tariffs.

17 In contrast, quantity data in the Comtrade database are all harmonized to kilograms.

the data, which are reported in table A1 in the appendix. Because it is crucial to use the same quantity unit for each product, we keep only the trade flows that use the most common quantity unit before aggregating the data to more coarse product definitions. The units used to measure quantity are very often the same, even within broad product categories. Approximately 1% of trade flow observations are dropped when harmonizing the quantity units at the 8-digit HS level. When harmonizing quantity units at the 6-digit and 4-digit HS level, we drop approximately 2% and 4% of observations, respectively.

As a robustness check, we perform our estimations using data from the Comtrade database, which is administered by the United Nations. We use importer-reported data for US imports at the 6-digit HS level for the years 1991 to 2015, where both the value of trade (in USD) and the quantity of trade (in kilograms) are reported.

To calculate the gains from trade for each imported product, we require data on import penetration ratios for each product, which we take from the US Bureau of Economic Analysis (BEA) 2007 input−output tables, available at the 6-digit level. We collapse the BEA commodity/industry classification to the 4-digit level, then merge it with the Center for International Data US import data at the 4-digit NAICS level.[18]

4.2. Empirical method

We estimate the lower and upper bounds of the elasticity of import demand for each good at the 4-, 6- and 8-digit HS level of aggregation, normalizing the variables as described above. Formally, the Leamer lower bound for good g, $\hat{\beta}_g$, is obtained directly by running the following regression:

$$\Delta^k \ln x_{gct} = -\hat{\beta}_g \Delta^k \ln p_{gct} + \varsigma_{gct}, \tag{24}$$

where

$$\Delta^k \ln x_{gct} \equiv \Delta \ln x_{gct} - \Delta \ln x_{gkt},$$
$$\Delta^k \ln p_{gct} \equiv \Delta \ln p_{gct} - \Delta \ln p_{gkt}.$$

The Leamer upper bound, $1/\hat{\beta}_g^R$, is obtained by running the following regression:

$$\Delta^k \ln p_{gct} = -\hat{\beta}_g^R \Delta^k \ln x_{gct} + \upsilon_{gct} \tag{25}$$

18 To assess the extent of measurement error in trade values and traded quantities due to human manipulation of the data, we test whether or not the data deviate from Benford's law. Benford's law describes the distribution of first digits in economic or accounting data. The results are reported in figure A1 in the appendix. We find that the distribution of first digits is very similar among the quantity and value data, which suggests that measurement error due to manipulation of the data is highly similar between quantities and values.

The lower bound based on trade values, $\hat{\beta}_g^P$, is found by running the following regression:

$$\Delta^k \ln v_{gct} = \left(1 - \hat{\beta}_g^P\right) \Delta^k \ln p_{gct} + \xi_{gct}, \tag{26}$$

where

$$\Delta^k \ln v_{gct} \equiv \Delta \ln v_{gct} - \Delta \ln v_{gkt}.$$

Finally, the upper bound based on trade values, $\hat{\beta}_g^{R,P}$, is found by running the following regression:

$$\Delta^k \ln p_{gct} = \left(\frac{1}{1 - \hat{\beta}_g^{R,P}}\right) \Delta^k \ln v_{gct} + \zeta_{gct} \tag{27}$$

4.3. Partial identification results

We first estimate the upper and lower bounds using the trade value−trade unit value specification as given by equations (26) and (27), which produces the bounds on the set of plausible point estimates based on quantity data. We call this set of possible estimates the "value-based bounds." The results for each 4-digit HS import product are illustrated in figure 3. The x-axis ranks each 4-digit HS product by its lower bound (least squares) estimate. While all lower bound estimates are positive and lie close to one, the estimates of the upper bound vary widely. For many products with a small lower bound estimate, the corresponding reverse least squares estimate is negative, which agrees with the predicted asymptotic bias. For several products the value-based upper bound is very high. We thus truncate the figure 3 to display estimates between 0 and 30. We also report all "value-based point estimates" based on trade values that the Soderbery (2015) procedure yields. The vast majority of the point estimates lie within the bounds given by the estimates of equations (26) and (27), with only a few exceptions.

We then estimate the point estimates and the upper and lower bounds using the Leamer trade quantity−trade unit value specification as given by equations (24) and (25), which we call the "Leamer bounds." We report both the Leamer bounds and the value-based bounds, plus the value-based point estimates, in figure 4. As predicted by the theory, the quantity-based and value-based lower bounds are identical, while the Leamer upper bound is far below the value-based upper bound. It is also evident that many of the value-based point estimates (around one third) lie above the Leamer upper bound. This suggests that many of the elasticity estimates used in the literature may be implausibly large. Finally, it is evident that the Leamer upper bounds are positive and lie above the lower bounds for all products, including those for which the value-based upper bound was negative.

We also check whether our results regarding the difference between the quantity-based and value-based upper bounds are sensitive to the level of

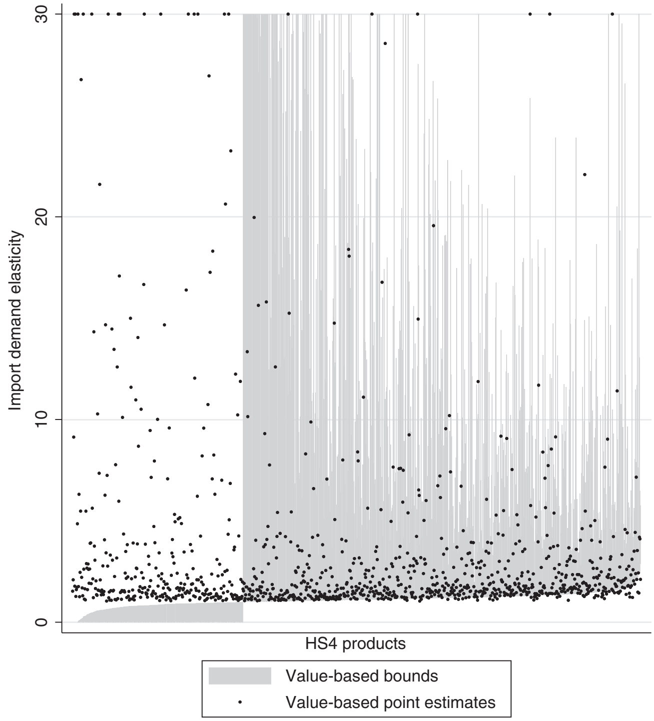

FIGURE 3 Value-based bounds and point estimates, by 4-digit Harmonized System, US, 1993–2006
NOTES: Each point estimate and bar corresponds to a unique 4-digit product. Products are ordered by the size of the lower bound estimate.
SOURCE: Center for International Data, authors' calculations

product aggregation. Imbs and Mejean (2015) show, for example, that estimates of trade elasticities are smaller in aggregate data than at finer levels of aggregation. In figure A2 in the appendix, we illustrate the alternative bounds with the original bounds and point estimates at the HS 6-digit level. We find that the difference between the Feenstra and Leamer upper bounds persists at finer levels of product aggregation. We also find that many of the value-based point estimates lie below the Feenstra and Leamer lower bounds even at finer levels of aggregation.

4.4. Point estimate results

We now turn to our point estimates of the import demand elasticities using quantity data and compare them with the point estimates derived from using trade value data, which is the standard approach in the literature. In figure 5, we illustrate the point estimates based on traded quantity data for each 4-digit HS import product, which we call the "quantity-based point estimates," as well as the corresponding value-based point estimate using trade value data. We also include the Leamer bounds, which allows us to discern how well the

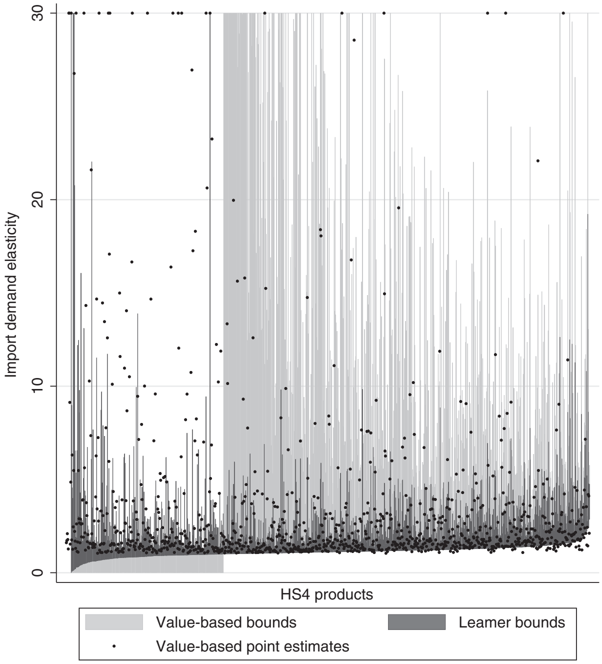

FIGURE 4 Leamer bounds, value-based bounds and point estimates, by 4-digit Harmonized System, US, 1993–2006

NOTES: Each point estimate and bar corresponds to a unique 4-digit product. Products are ordered by the size of the lower bound estimate.

SOURCE: Center for International Data, authors' calculations

point estimates fit within the set of plausible estimates. Figure 5 illustrates that the value-based point estimates tend to be larger than the quantity-based point estimates on average. The raw correlation between the quantity-based and value-based point estimates is 0.14.[19]

Descriptive statistics of all of the bounds and point estimates at the 4-, 6- and 8-digit level are provided in table 2, where we report the number of products, the raw mean and the median. The mean and median of the Leamer upper bounds are always lower than the corresponding measure of the value-based upper bounds, regardless of the level of product aggregation. The median is lower than the mean in all cases for the upper bounds, which is driven by a small number of products with relatively high upper bounds. The difference between the mean and the median is especially pronounced for the value-based upper bounds. Table 2 also highlights that the quantity-based point estimates are lower than the value-based point estimates in all cases,

19 As a robustness check, we estimate the quantity-based point estimates, the value-based point estimates and the Leamer bounds using data from the COMTRADE database at the 4-digit HS level. The results, illustrated in figure A3 in the appendix, are very similar to those illustrated in figure 5.

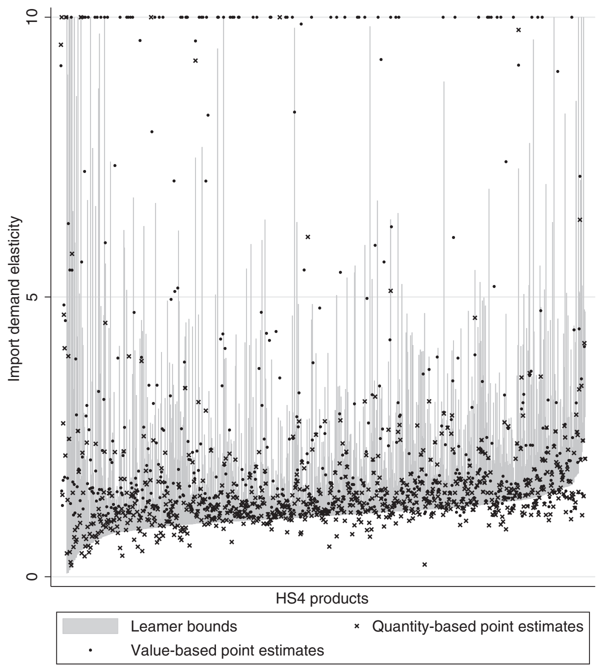

FIGURE 5 Value-based point estimates, quantity-based point estimates and Leamer bounds by 4-digit Harmonized System, US, 1993–2006

NOTES: Each point estimate and bar corresponds to a unique 4-digit product. Products are ordered by the size of the lower bound estimate.

SOURCE: Center for International Data, authors' calculations

for all levels of product aggregation. The raw average and median of the point estimates are very stable across product aggregations.

5. Implications for the gains from trade

We now quantify the economic importance of our alternative approach to measuring import demand elasticities for the welfare gains from economic integration. We use the framework developed by Arkolakis et al. (2012) and adapted to the multi-sector framework by Ossa (2015) and Costinot and Rodríguez-Clare (2014), which distill the welfare gains from trade compared to autarky across a wide array of trade models into a simple formula:

$$\hat{G}_j = 1 - \prod_{s=1}^{S} \left(\hat{\lambda}_{jj,s} \frac{e_{j,s}}{r_{j,s}} \right)^{\beta_{j,s}/\varepsilon_s}, \tag{28}$$

where $\hat{G}_j$ is the percentage change in welfare in destination country j when moving from the status quo to autarky, $\hat{\lambda}_{jj,s}$ equals the percentage change in country j's internal trade in sector s (1 minus the import penetration ratio), $e_{j,s}$ denotes the share of total expenditure in country j allocated to sector s, and $\beta_{j,s} = e_{j,s}$ assuming Cobb–Douglas preferences between sectors. $r_{j,s}$ denotes the share of total revenues in country j generated from sector s, and ε_s

TABLE 2
US import elasticity descriptive statistics

	Quantity-based point estimate	Leamer lower bound	Leamer upper bound	Value-based point estimate	Value-based lower bound	Value-based upper bound
Panel A: 4-digit HS						
Count	674	674	674	674	674	447
Mean	1.55	1.09	3.29	7.45	1.09	39.5
Median	1.31	1.08	2.39	1.62	1.08	10.2
Panel B: 6-digit HS						
Count	2,775	2,775	2,775	2775	2,775	1,779
Mean	2.26	1.14	5.87	6.59	1.14	96.3
Median	1.42	1.11	2.95	1.73	1.11	10.2
Panel C: 8-digit HS						
Count	4,847	4,847	4,847	4,847	4,847	3,034
Mean	2.60	1.14	12.2	6.44	1.14	80.7
Median	1.47	1.09	3.37	1.81	1.09	10.8

NOTE: The sample is restricted to those products for which value- and quantity-based point estimates exist.
SOURCE: Center for International Data, authors' calculations

is the elasticity of imports with respect to variable trade costs in sector s, also known as the "trade elasticity." In the Armington (1969) model, $\varepsilon = 1 - \sigma$, where σ is the import demand elasticity.[20] The formula given in equation (28) thus highlights that estimates of the import demand elasticity play a central roll in measuring the gains from trade.

We first calculate the point estimates at the 4-digit BEA commodity classification level. These estimations yield 50 BEA commodities for which we have viable value-based and quantity-based point estimates, and we illustrate these estimates in figure A4 in the appendix. We then combine these point estimates with data on the import penetration ratio from the 2007 BEA input−output tables and calculate the gains from trade following equation (28). We find that the overall gains from trade are 48% using point estimates based on traded quantity data versus 24% using point estimates based on trade value data.[21]

An alternative approach is to gauge the gains from trade using the upper bounds on the import demand elasticities instead of the point estimates in equation (28). Using the upper bounds instead of point estimates yields more conservative gains, but the difference between value data or quantity data remains large. Using the quantity-based upper bounds yields a 30% overall gain, while using the value-based upper bounds yields a 6% overall gain.

20 In the Melitz (2003) model, $\varepsilon = 1 - \sigma - \gamma_j$, where γ_j is the extensive margin elasticity. In the Ricardian model, $\varepsilon = 1 - \sigma + \gamma^i_{jj} - \gamma^i_{ij}$, where γ^i_{jj} and γ^i_{ij} denote the extensive margin elasticities.

21 Our 24% estimate using trade value data is comparable with Ossa's (2015) estimate of a 19% gain from trade for the United States. The differences between our estimates are likely driven by the fact that Ossa (2015) estimates import demand elasticities on a panel of 129 countries using data from GTAP.

Overall, the lower import demand elasticities obtained using quantities translate into much larger gains from trade compared to the estimates obtained using trade value data.

6. Conclusion

Accurate estimates of import demand elasticities are essential for measuring the gains from trade and predicting the impact of trade policies. The international economics literature has typically estimated these elasticities using trade value data instead of trade quantities. Using partial identification methods, we show theoretically that the upper bound on the import demand elasticity is more biased upward compared to using traded quantity data. We confirm our theoretical predictions using detailed US import data. We also generate import demand elasticity point estimates based on traded quantity data and compare them with corresponding point estimates using trade value data. Our results suggest that import demand elasticities are lower than previously thought for many goods, which implies that the gains from economic integration have been underestimated in earlier studies.

While we test and motivate our analysis in the context of international trade, our results are generalizable to any estimation of demand elasticities where price data must be constructed from quantity and value data, and the econometrician must select the most appropriate model. Our derivations of the asymptotic bias suggest that using quantity data is superior to using value data in cases where measurement error is low or is of similar magnitude in the quantity and value data.

Our theoretical results suggest that the relative size of measurement error in the trade value and trade quantity data is crucial in order to determine how to correctly estimate import demand elasticities. While we touch on the major issues here, a formal assessment of measurement error in trade value and trade quantity data would be a valuable topic for future empirical research. We argue that measurement error is a smaller problem in the US import data that we use here, but measurement error may be a larger issue in import data from developing countries.

Our results have many implications in international economics that we leave for further research, such as analyzing the impact on the variety gains from trade or the magnitude of trade costs implied by trade flow data. Given that these elasticities are so important for understanding the gains from trade, it is hoped that our study encourages discussion on the pros and cons of using quantity versus value data when estimating demand elasticities.

Appendix A.1. Deviations from Benford's law in the traded quantity and trade value data

In order to access the extent of measurement error in trade values and traded quantities due to human manipulation of the data, we test whether the data deviate from Benford's law. Benford's law describes the distribution of first digits in economic or accounting data. For each 10-digit product,

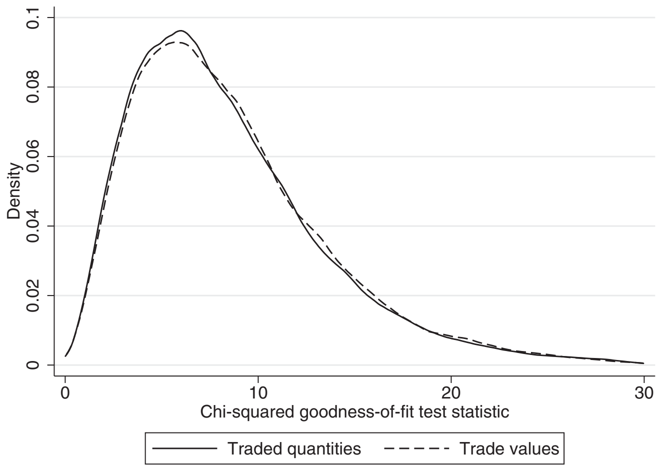

FIGURE A1 Deviations from Benford's law in traded quantity and trade value data, by 10-digit Harmonized System, US, 1993−2006

SOURCE: Center for International Data, authors' calculations

TABLE A1

Units for quantity

Abbreviation	Description	Abbreviation	Description
BBL	Barrel	KG	Kilograms
CAR	Carat	KM3	Thousand cubic metres
CBM	Cubic metres	LNM	Linear metres
CGM	Silver/gold content in grams	LTR	Litres
CKG	Content kilogram	M2	Square metres
CM2	Square centimetres	MBQ	Megabecquerels
CTN	Content metric ton	MC	Millicurie
CUR	Curie	MTR	Metre
CYK	Clean yield kilogram	MWH	Megawatt hour
DOZ	Dozen	NO	Number
DPC	Dozen pieces	PCS	Pieces
DPR	Dozen pair	PFL	Proof litre
DS	Doses	PK	Packs
FBM	Fibre metre	PRS	Pairs
GBQ	Gigabecquerels	SQ	Square
GCN	Gross containers	THM	Thousand metres
GKG	Kilogram (gross)	THS	Thousands
GM	Gram	TNV	Ton raw value (metric)
GRS	Gross	TON	Metric tons
HUN	Hundred		

NOTES: This table lists the quantity units used in the analysis. We consolidate abbreviations with identical descriptions. We rewrite HND as HUN (hundred), L as LTR (litres), M as MTR (metres), PKS as PK (packs), CBM as M3 (cubic metres), TCM as KM3 (thousand cubic metres) and T as TON (metric tons).

the goodness-of-fit test statistic is calculated using product-level export data according to the following formula:

$$N \sum_{d=1}^{9} \frac{\left(f^d - \hat{f}^d\right)^2}{f^d}, \tag{A1}$$

where $\hat{f}^d$ is the fraction of digit d in the data and f^d is the fraction predicted by Benford's law. The test statistic converges to a χ^2 distribution with eight degrees of freedom as N approaches infinity. The corresponding 10%, 5% and 1% critical values are 13.4, 15.5 and 20.1, respectively.

The distributions of the χ^2 goodness-of-fit test statistic values for the US import value and import quantity are illustrated in figure A1. We find that the distribution of first digits is very similar among the quantity and value data, which suggests that measurement error consistent with manipulation of the data is highly similar between quantities and values.

Appendix A.2. Partial identification using the quantity−value approach

As suggested by Scobie and Johnson (1975), another way to estimate import demand elasticities is regress $\ln x_{ct}$ on $\ln v_{ct}$, thus avoiding the need to construct price data. We again use equation (8) to transform equations (1) and (2) into a regression of trade quantities on trade values. The regression equation is

$$\ln x_{ct} = \frac{-\beta}{1-\beta} \ln v_{ct} + \frac{1}{1-\beta} \varepsilon_{ct}, \tag{A2}$$

where we denote the OLS coefficient $\hat{\delta}^X$. We define $\hat{\delta}^V$ as the coefficient from the reverse regression of $\ln v_{ct}$ on $\ln x_{ct}$:

$$\ln x_{ct} = \frac{\gamma}{1+\gamma} \ln v_{ct} + \frac{1}{1+\gamma} \eta_{ct} \tag{A3}$$

The corresponding estimates of β are

$$\hat{\beta}^X = \frac{\hat{\delta}^X}{\hat{\delta}^X - 1}, \tag{A4}$$

$$\hat{\beta}^V = \frac{1}{1-\hat{\delta}^V}. \tag{A5}$$

The reduced form is given by

$$\ln v_{ct} = \frac{1+\gamma}{\gamma+\beta} \varepsilon_{ct} - \frac{1-\beta}{\gamma+\beta} \eta_{ct},$$

$$\ln x_{ct} = \frac{\gamma}{\gamma+\beta} \varepsilon_{ct} + \frac{\beta}{\gamma+\beta} \eta_{ct}.$$

The probability limit of the OLS estimates of δ^X and δ^V are thus

$$\texttt{plim}\ \hat{\delta}^X = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2}{(1+\gamma)^2\sigma_\varepsilon^2 + (1-\beta)^2\sigma_\eta^2}, \tag{A6}$$

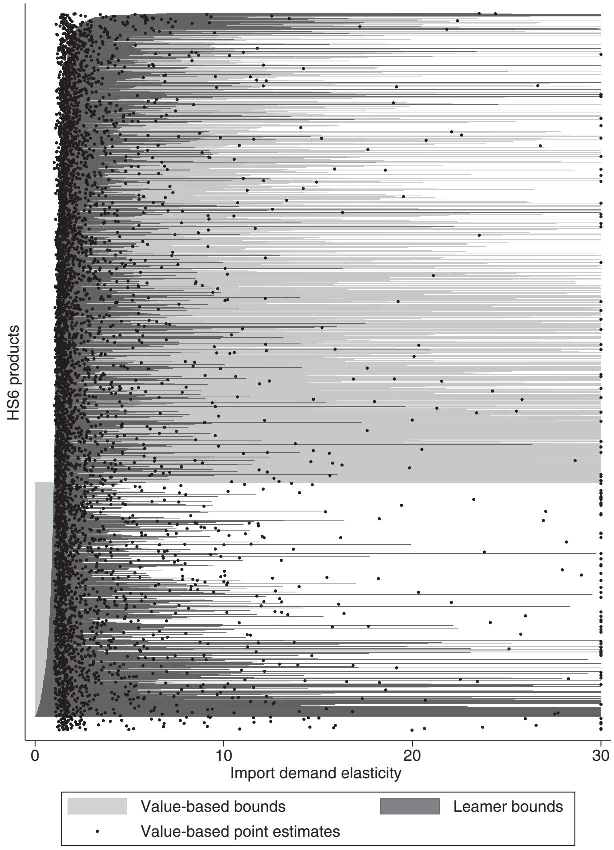

FIGURE A2 Leamer bounds, Feenstra bounds and point estimates, by 6-digit Harmonized System, US, 1993–2006
NOTES: Each point estimate and bar corresponds to a unique 6-digit product. Products are ordered by the size of the lower bound estimate.
SOURCE: Center for International Data, authors' calculations

$$\texttt{plim } \hat{\delta}^V = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2}{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2}. \tag{A7}$$

These direct OLS estimates, when expressed in terms of β^X and β^V, are

$$\texttt{plim } \hat{\beta}^X = \beta + (\beta+\gamma)\frac{(1+\gamma)\sigma_\varepsilon^2}{(\beta-1)\sigma_\eta^2 - (1+\gamma)\sigma_\varepsilon^2}, \tag{A8}$$

$$\texttt{plim } \hat{\beta}^V = \beta + (\gamma+\beta)\frac{\gamma\sigma_\varepsilon^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2} \lesseqgtr \beta. \tag{A9}$$

The probability limit of the lower bound in this case is equivalent to the Leamer upper bound, while the probability limit of the upper bound is equivalent to the value-based upper bound. It follows that the quantity-value lower bound will not hold if the Leamer upper bound holds. It also follows that the union of the Leamer and quantity-value bounds is equal to the value-based bounds.

Regressing trade quantities on trade values tends to overestimate the lower bound. In the vast majority of cases where the Leamer upper bound parameter restrictions are met, this implies that the parameter assumptions required for the quantity−value lower bound to hold are unlikely to be met.

Appendix A.3. Quantity−value approach with measurement error

When regressing traded quantities on trade values, the probability limit of the OLS estimates of δ^X and δ^V are

$$\texttt{plim}\ \hat{\delta}^X = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2 + (\gamma+\beta)^2\sigma_{uw}}{(1+\gamma)^2\sigma_\varepsilon^2 + (1-\beta)^2\sigma_\eta^2 + (\gamma+\beta)^2\sigma_u^2},$$

$$\texttt{plim}\ \hat{\delta}^V = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2 + (\gamma+\beta)^2\sigma_{uw}}{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\gamma+\beta)^2\sigma_w^2}.$$

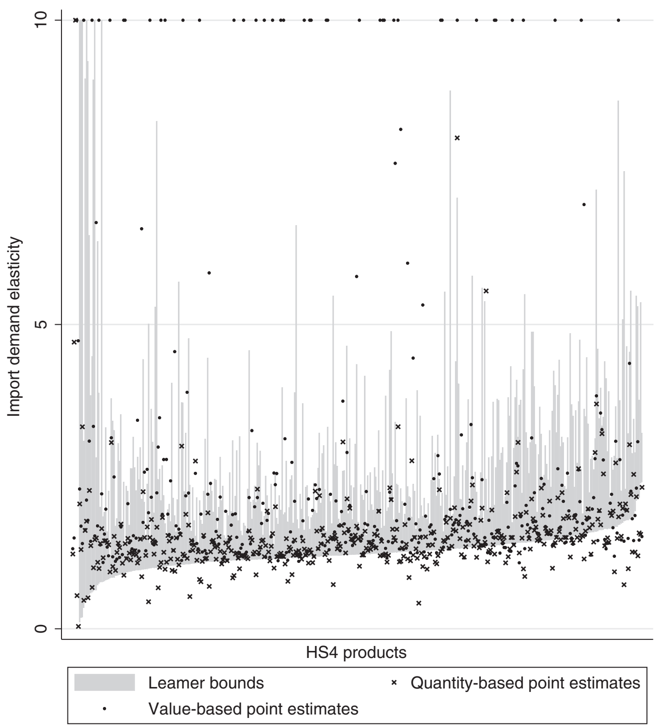

FIGURE A3 Value-based point estimates, quantity-based point estimates and Leamer bounds by 4-digit Harmonized System, US, 1991−2015

NOTES: Each point estimate and bar corresponds to a unique 4-digit product. Products are ordered by the size of the lower bound estimate.

SOURCE: COMTRADE, authors' calculations

These direct OLS estimates, when expressed in terms of β^X and β^V, yield probability limits equal to equations (14) and (15), respectively:

$$\texttt{plim}\ \hat{\beta}^X = \beta + (\beta+\gamma)\frac{(1+\gamma)\sigma_\varepsilon^2 + (\beta+\gamma)\sigma_u^2 + (\beta-1)(\beta+\gamma)\left(\sigma_u^2 - \sigma_{uw}\right)}{(\beta-1)\sigma_\eta^2 - (1+\gamma)\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_u^2 - \sigma_{uw}\right)} \tag{A10}$$

$$\texttt{plim}\ \hat{\beta}^V = \beta + (\beta+\gamma)\frac{\gamma\sigma_\varepsilon^2 + \beta(\beta+\gamma)\left(\sigma_{uw} - \sigma_w^2\right) + (\beta+\gamma)\sigma_w^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 - (\beta+\gamma)^2\left(\sigma_{uw} - \sigma_w^2\right)} \tag{A11}$$

Appendix A.4. Additional tables and figures

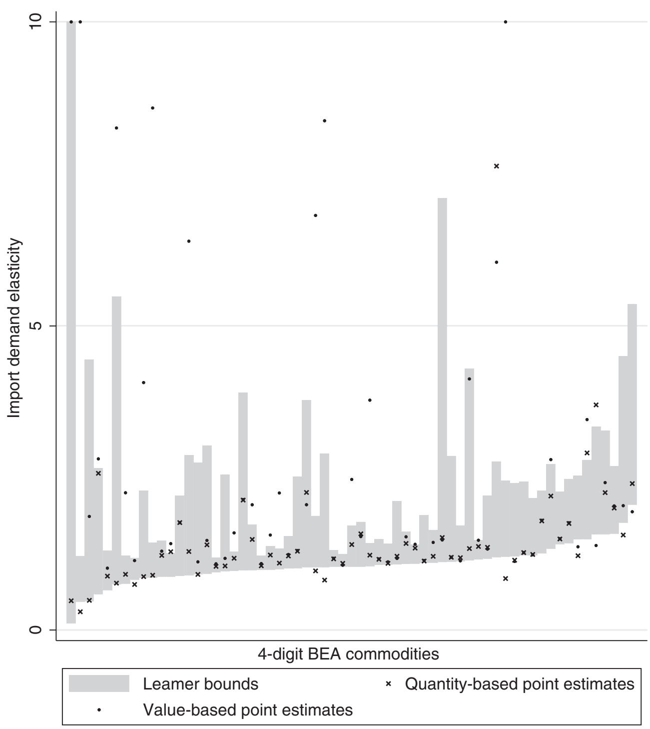

FIGURE A4 Value-based point estimates, quantity-based point estimates and Leamer bounds by 4-digit US Bureau of Economic Analysis (BEA) commodity, US, 1993–2006

NOTES: Each point estimate and bar corresponds to a unique 4-digit product. Products are ordered by the size of the lower bound estimate.

SOURCE: Center for International Data and BEA, authors' calculations

Supporting information

The data and code that support the findings of this study are available in the Canadian Journal of Economics Dataverse at `https://doi.org/10.5683/SP3/CDE1YV`.

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